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DOR(J&L) RESEARCH NOTE RN 2000/16

EVALUATION OF A WAR GAMING AND COMBAT MODELLING COURSE

BY

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JULY 2000

OTTAWA, CANADA

National Defence Défense nationale

OPERATIONAL RESEARCH DIVISION

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Jason Offiong

Recommended by P Comeau L/SPORT

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OTTAWA, ONTARIO JULY 2000

ABSTRACT

This author attended the War Gaming and Combat Modelling Course from 15 to 19 May 2000. The course is offered by the Applied Math and Operational Research Group at the Royal Military College of Science in Shrivenham, UK. The course syllabus included the deterministic Lanchester Square and Linear Laws, a stochastic Lanchester approach to simulation, measures of effectiveness, exercises for the students, a presentation of several combat models in use or under development today in the UK. This course was an excellent introduction to the fields of war gaming and combat modelling. It is recommended that all junior defence scientists be encouraged to attend this course.

TABLE OF CONTENTS

EVALUATION OF A WAR GAMING AND COMBAT MODELLING COURSE

I. GENERAL DESCRIPTION

1. A War Gaming and Combat Modelling Course (WGCMC) was conducted by the Applied Math and Operational Research Group (AMOR) in the Department of Informatics and Simulation at the Royal Military College of Science (RMCS) at Shrivenham, England. The duration of the course was five days and it consisted of lectures, problem solving exercises and practical sessions. Presently, AMOR runs the WGCMC twice a year. This Research Note (RN) discusses the course conducted from 15 – 19 May 2000.

2. The majority of the students in the course are part-time master's students in Military Operational Research at AMOR. This course is part of their required curriculum.

Aim

3. The aim of the course was to provide participants with an understanding of how war games, combat simulations and analytic battle models are used to represent combat on land, sea or in the air for the training, testing and assessment of military forces and equipment. This RN is intended to summarise the key concepts presented on the course, provide a brief introduction to the models discussed there, and assess the value of the course to Operational Research Division (ORD) defence scientists.

Programme

4. The WGCMC was organised into six main study areas outlined as follows:

   a. Introduction. An introduction to the methods used in modelling combat and their application in support of defence decision-making and training.

   b. Combat Simulation. The basic principles of discrete event Monte Carlo simulations of combat were illustrated through the use of a simple engagement model. The concepts were then extended to allow the representation of a more realistic battlefield. Aggregated models of

combat were discussed, as was the application of system dynamics to the modelling of combat.

c. War Gaming / Interactive Simulation. The underlying principles of war gaming and the interactive simulation of combat as used for the assessment, testing and training of military forces and their equipment were covered. Participants were shown constructive, virtual, live simulations of combat and the synthetic battlefield.

d. Lanchester's Equations. The deterministic and stochastic Lanchester equations for direct and indirect fire as used for both homogeneous and heterogeneous forces were covered, as was the application of Lanchester's equations in current models of combat.

e. War Gaming and Combat Modelling in Practice. Consideration was given to the following issues related to the practical application of war gaming and combat modelling:

(1) Data and scenarios;

(2) Terrain modelling;

(3) Combat algorithms (attrition and movement);

(4) The representation of human factors;

(5) Measures of effectiveness;

(6) The verification and validation of combat models;

(7) Automated forces;

(8) Simulation for training; and

(9) Distributed interactive simulation.

f. Practical Sessions. Throughout the course, reference was made to existing models of combat and the facilities of the Simulation and Synthetic

Environment Laboratory (SSEL). There was an opportunity for all course members to gain "hands-on" experience of several of the models.

Prerequisites

5. Participants should ideally have a degree in a quantitative discipline and some knowledge of calculus. An understanding of the basic principles of Monte Carlo simulation would be helpful, but is not essential.

II. INSTRUCTION

6. The WGCMC began with an introduction to War Gaming and Combat Modelling. This discussion set the context of how war gaming fits within the broader field of strategic analysis. Combat modelling was described as "the representation of the interactions that occur between opposing forces and their weapon platforms operating with known tactics in a known environment" [1].

7. Two different model categories were presented.

a. Deterministic Models. Each run of a deterministic model will produce identical results if the given set of input data remains unchanged. A deterministic model can be used to analyse the "average" or expected outcome of a particular engagement or simulation.

b. Probabilistic or Stochastic Models. Each run of the discrete event simulation model will produce only one of the many possible outcomes of the process for the given set of input data. Many iterations of a stochastic model are required to develop a statistically significant output data set.

The Fundamental Duel and The Lanchester 'Square' Law

8. The Fundamental Duel is the simplest description of a combat situation. It is a one-versus-one engagement between two weapon platforms, Blue and Red, which exchange direct fire. Each side has some inter-firing time distribution and a single shot kill probability (SSKP). The duel continues until one of the combatants is killed.

9. The outcome of the Fundamental Duel is solved using the Lanchester Square Law. F. W. Lanchester was a British engineer who in 1914 published a paper describing a model of attrition in battle. Lanchester made the following assumptions:

a. Two forces attack each other. Each unit on each side is within weapon range of all units on the other side;

b. Units on each side are identical, but the units on one side may have a different killing rate to the opposing units;

c. Each firing unit is sufficiently well aware of the location and condition of all enemy units so that when a target is killed, fire may immediately be shifted to a new target; and

d. Fire is uniformly distributed over surviving units.

10. The original model was a deterministic mathematical representation of the attrition process in combat based on a pair of linked differential equations [1]:

$$\begin{aligned} \frac{db}{dt} &= -\rho r \\ \frac{dr}{dt} &= -\beta b \end{aligned} \tag{1}$$

11. In these equations,

a. b is the number of Blue units remaining at time t ($b = B$ at $t = 0$);

b. r is the number of Red units remaining at time t ($r = R$ at $t = 0$);

c. β is the rate at which a single Blue unit kill Red units; and

d. ρ is the rate at which a single Red unit kill Blue units.

12. The time independent solution of this system is given by:

$$\beta\left(B^2 - b^2\right) = \rho\left(R^2 - r^2\right) \tag{2}$$

The two kill rates, β and ρ, are related to the weapon characteristics of each side. If p_b is the single shot probability that a Blue weapon will kill a Red unit and f_b is the rate of fire of each Blue weapon, then

$$\beta = p_b f_b \tag{3}$$

and similarly,

$$\rho = p_r f_r \tag{4}$$

13. The solution, equation (2), shows the inherent value of one of the Principals of War: *Concentration of Force*, because of its dependence on the squares of the quantities of units. As Field Marshall Slim put it: "The more you use, the fewer you lose."

14. The outcome of a Lanchester Fundamental Duel is readily determined according to the following equations:

$$\begin{aligned} \beta b^2 - \rho r^2 > 0 \quad &\Rightarrow \quad \textit{Blue Win} \\ \beta b^2 - \rho r^2 < 0 \quad &\Rightarrow \quad \textit{Red Win} \\ \beta b^2 - \rho r^2 = 0 \quad &\Rightarrow \quad \textit{Parity} \end{aligned} \tag{5}$$

15. A time-dependent solution can be found as well to give the number of Blue and Red units remaining at any time:

$$\begin{aligned} b &= B\cosh\left(\sqrt{\beta\rho}\,t\right) - R\sqrt{\frac{\rho}{\beta}}\sinh\left(\sqrt{\beta\rho}\,t\right) \\ r &= R\cosh\left(\sqrt{\beta\rho}\,t\right) - B\sqrt{\frac{\beta}{\rho}}\sinh\left(\sqrt{\beta\rho}\,t\right) \end{aligned} \tag{6}$$

16. The differential equations (1) can readily be expanded to more complex battlefield situations. For instance, a direct fire confrontation with two types of weapon platforms on each of the Red and Blue sides is described by:

$$
\begin{aligned}
-\frac{db_1}{dt} &= \rho_{11} r_1 \frac{b_1}{b_1 + b_2} + \rho_{21} r_2 \frac{b_1}{b_1 + b_2} \\
-\frac{db_2}{dt} &= \rho_{12} r_1 \frac{b_2}{b_1 + b_2} + \rho_{22} r_2 \frac{b_2}{b_1 + b_2} \\
-\frac{dr_1}{dt} &= \beta_{11} b_1 \frac{r_1}{r_1 + r_2} + \beta_{21} b_2 \frac{r_1}{r_1 + r_2} \\
-\frac{dr_1}{dt} &= \beta_{12} b_1 \frac{r_2}{r_1 + r_2} + \beta_{22} b_2 \frac{r_2}{r_1 + r_2}
\end{aligned}
\tag{7}
$$

The kill rates, β_{xy}, represent the rates at which a single Blue weapon-type x kill Red units of type y. Again, a similar formulation for ρ_{wz} exists.

Indirect-Fire and the Lanchester 'Linear' Law

17. A Lanchester model can also be applied to indirect fire weapons. The same assumptions as in paragraph 9 are valid, except the last one which now states that fire from surviving units is distributed uniformly over the area in which enemy forces are located.

18. The notation for modelling indirect fire is as follows:

a. A_r, A_b = areas in which the Red and Blue forces are located;

b. A_{er}, A_{eb} = areas of effectiveness of a single shot from Red and Blue; and

c. f_r, f_b = the rate of fire of Red and Blue weapons.

19. Now the kill rates for each side can be defined as:

$$
\begin{aligned}
K_r &= \frac{f_r A_{er}}{A_b} \\
K_b &= \frac{f_b A_{eb}}{A_r}
\end{aligned}
\tag{8}
$$

Then, the Lanchester Differential Equations are

$$\frac{dr}{dt} = -K_b br \quad and$$
$$\frac{db}{dt} = -K_r br \tag{9}$$

with solution:

$$K_b(B-b) = K_r(R-r) \tag{10}$$

20. Note that in situations described by the Linear Law, there is no distinct advantage in the concentration of forces.

21. The square and linear laws can be used together to model mixed force battles which include both direct and indirect fire weapon systems. For example, if both Blue and Red sides have a combination of infantry and artillery and we assume that each side's artillery units can fire at both the opposition's infantry and artillery but that the infantry forces can only engage one another. Then, ϕ can be defined as the fraction of Red's artillery rounds used for counter-battery and hence $1\text{-}\phi$ is the fraction used against Blue's infantry. Similarly for Blue, φ is defined.

22. Then, the differential equations describing this engagement are:

$$\begin{aligned}
\frac{db_1}{dt} &= -(1-\phi)K_{RI}b_1r_2 - \rho r_1 \\
\frac{db_2}{dt} &= -\phi K_{RA}b_2r_2 \\
\frac{dr_1}{dt} &= -(1-\varphi)K_{BI}r_1b_2 - \beta b_1 \\
\frac{dr_2}{dt} &= -\varphi K_{BA}r_2b_2
\end{aligned} \tag{11}$$

23. In equations (11), the following new notation has been introduced:

a. b_1, r_1 are the number of Blue/Red infantry units remaining at time t;

b. b_2, r_2 are the number of Blue/Red artillery units remaining at time t; and

c. K_{xy} is the kill rate for side x when engaging the opposing forces y units (I ≡ infantry, A ≡ artillery).

24. Using the Lanchester Square and Linear Laws, very complex battles can be modelled. The Lanchester equations can also be expanded to consider factors such as reinforcement of forces, distance between opposing units, the reduction of losses due to speed of retreat and even small force guerrilla engagements.

A Stochastic Approach

25. An equivalent stochastic formulation of the Lanchester concept for a direct-fire battle was also presented. This stochastic method allows the probabilistic outcomes of a battle to be investigated. Once again, similar notation and assumptions as in the deterministic Lanchester Square model are used.

26. The probability that at some time, t, after the start of a battle, there are b Blue units and r Red units remaining is denoted as $P(b, r, t)$. The approach to determine $P(b, r, t)$ is to first find $P(b, r, t+\delta t)$ and then let $\delta t \rightarrow 0$ so that the probability that there will be more than one kill in that time is negligible.

27. The probability that a Blue force of size b will obtain a Red kill in time δt is $\beta b \delta t$. Similarly the probability that Red unit will kill a Blue unit is $\rho r \delta t$.

28. Thus, $P(b, r, t+\delta t)$ is given by:

$$\begin{aligned} P(b,r,t+\delta t) = {} & P(b,r,t)\cdot[1-\beta b\delta t][1-\rho r\delta t] \\ & + P(b+1,r,t)\cdot[\rho r\delta t][1-\beta b\delta t] \\ & + P(b,r+1,t)\cdot[1-\rho r\delta t][\beta b\delta t] \end{aligned} \tag{12}$$

If terms in δt^2 are ignored, this simplifies to:

$$\begin{aligned} \frac{P(b,r,t+\delta t)-P(b,r,t)}{\delta t} = {} & -(\rho r+\beta b)P(b,r,t) \\ & +(\rho r)P(b+1,r,t) \\ & +(\beta b)P(b,r+1,t) \end{aligned} \tag{13}$$

The left side of equation (13) is simply the time derivative of $P(b, r, t)$, so

$$\frac{\partial P(b,r,t)}{\partial t} = -(\rho r + \beta b)P(b,r,t) + (\rho r)P(b+1,r,t) + (\beta b)P(b,r+1,t) \qquad (14)$$

29. Using a similar analysis, it can be shown that

$$\frac{\partial P(b,0,t)}{\partial t} = (\beta b)P(b,1,t), \qquad (15)$$

$$\frac{\partial P(0,r,t)}{\partial t} = (\rho r)P(1,r,t) \text{ and} \qquad (16)$$

$$\frac{\partial P(B,R,t)}{\partial t} = -P(B,R,t)\cdot[1-\beta B\delta t][1-\rho R\delta t] \qquad (17)$$

This last expression may be integrated to give

$$P(B,R,t) = e^{-(\beta B+\rho R)t} \qquad (18)$$

which is the probability that no units from either side will be have been killed by time t.

30. There are a number of boundary conditions for this stochastic formulation:

$$\begin{aligned} &P(B,R,0) = 1 \\ &P(b,r,0) = 0 \quad for\, b < B, r < R \\ &P(b,r,t) = 0 \quad for\, b > B, b < 0, r > R, r < 0 \end{aligned} \qquad (19)$$

31. The system of equations (14) through (16) and (18) with the boundary conditions (19) may be solved iteratively to give for any battle state *(b, r)*, at time t the probability *P(b, r, t)*.

32. Equation (18) can be used as the basis of a stochastic battle simulation. Specifically, it can give the probability that another battle kill will have occurred by some time, Δt, after the previous battle kill. If the previous kill occurred at time t, and the battle state was given at t by *(b, r)* then by time $t+\Delta t$, the probability that the battle state will be either *(b-1, r)* or *(b, r-1)* is given by

$$1 - e^{-(\beta b+\rho r)t} \qquad (20)$$

From this expression, it can be seen that the times between battle kills are negatively exponentially distributed and the mean time to the next kill for a battle at state *(b, r)* is

$$\frac{1}{\beta b + \rho r} \tag{21}$$

33. Whenever a battle kill occurs, it must be determined whether it was a Blue or a Red unit that was destroyed. It is stated above that the probability that the Blue force will obtain a Red kill in time δt is $\beta b \delta t$ and similarly the probability that Red unit will kill a Blue unit is $\rho r \delta t$. Hence, the conditional probability that given that a kill has occurred at battle state *(b, r)*, the probability that it is a Red unit is:

$$\frac{\beta b}{\beta b + \rho r} \tag{22a}$$

and the probability that it was a Blue unit is:

$$\frac{\rho r}{\beta b + \rho r} \tag{22b}$$

34. So, equations (21) and (22) enable a simple simulation approach to be adopted.

a. A negative exponential distribution with mean $\frac{1}{\beta b + \rho r}$ is sampled to give the length of time the battle is in state *(b, r)* before the transition to *(b-1, r)* or *(b, r-1)*.

b. Determine whether a Blue or a Red unit has been killed by sampling the distribution given by equations (22a).

c. By repeating steps a and b, the times and details of each kill can be tracked starting at *(B, R)* and $t = 0$ until some desired end point.

35. A number of such battles as described in paragraph 34 are required to generate statistically significant battle data. This simulation approach may be easily adapted to cope with more complex battle situations, for instance, heterogeneous forces that require weapon to target allocation rules.

36. The number of cases that have been studied by means of the Lanchester approach and verified by means of actually observed data is small. However, the usefulness of the model is in its capacity for quickly providing a comparative evaluation of several possible approaches and bringing to the forefront the underlying facts and principles rather than obtaining solutions to specific problems.

Measures of Effectiveness (MoE)

37. Many measures of a force's effectiveness (MoE) can be defined. Which measure is most appropriate depends on the specific application. Although many more are possible, a list of sample MoE's is given:

a. Difference in Losses: $[R - r(t)] - [B - b(t)]$;

b. Loss Rate Difference: $\frac{R - r(t)}{R} - \frac{B - b(t)}{B}$;

c. Ratio of Percent Losses: $\frac{\frac{R - r(t)}{R}}{\frac{B - b(t)}{B}}$;

d. Surviving Force Ratio: $\frac{b(t)}{r(t)}$;

e. Loss Exchange Ratio: $\frac{R - r(t)}{B - b(t)}$;

f. Time required for attacker to reach a terrain objective;

g. Attacker loss per kilometre advanced; and

h. Area seized by attacker per attacker lost.

Exercises

38. These concepts were reinforced to the students on the WGCMC through the use of several exercises. The students completed these exercises individually or in small groups. These exercises and their solutions are included in the Annex A, Appendices 1 through 4.

War Gaming Models

39. Once the theory shown in the first half of this section was introduced, the remainder of the WGCMC focussed on the presentation of various war games and combat models currently in use. For most of the models, one or more lectures were given and then a practical session was scheduled. These practical sessions gave the student hands-on experience and were very well received.

40. Some of the models that were presented are used for training and education, while others are actually used for operational analysis. Throughout the WGCMC, this distinction was often stressed. Figure 1 shows how these two distinct uses of combat models are broken down and some uses of each.

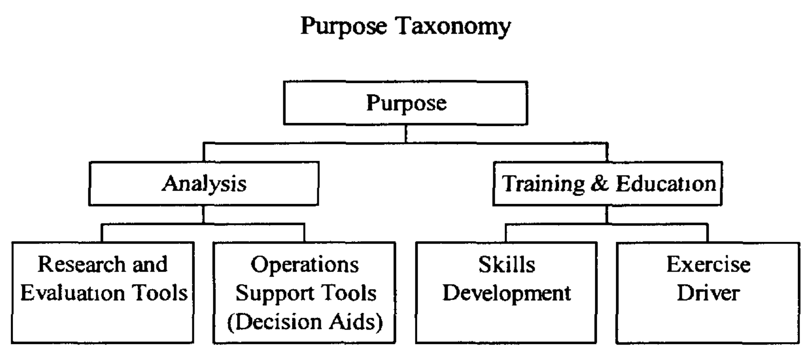

Figure 1 – Purpose and usage of combat models [2].

41. The detailed difference between training and analysis models is summarised in Table I.

42. The discussion of combat models began with an introduction to computer generated forces (CGF). CGF is defined as "A realistic representation of the weapons, platforms and people that form part of a rich operational environment. The Order of Battle (ORBAT), characteristics and behaviours are simulated, including the decision making of commanders and their staff" [3].

TABLE I
COMPARISON BETWEEN TRAINING COMBAT MODELS AND ANALYSIS COMBAT MODELS [4]

	Training Model	Analysis Model
Objective	Participant's Involvement	Explore system / issues
Outcome	Known in advance	To be discovered
Players	Primary focus of system	An enabling component
Outputs	Knowledge and experience	Understanding and data
Play Speed	Critical (real-time or Very near real-time)	Not necessarily critical
Usage	Often frequent and repetitive	Specific and varied
Key Factors	• Plausibility • Transfer of Training • Player Interaction	• Degree of rigour • Audit trails • Adaptability

43. The remainder of this section provides a brief overview of the key models that were instructed on the WGCMC.

Janus

44. The US Army developed the Janus war game in the late 1970's. This model is an interactive, six-sided, closed ground/air combat simulation [5]. Janus is an event-driven stochastic model. Janus is the classical example of a "human-in-the-loop" (HITL) war game. It requires all moves be explicitly mapped by human players and the computer system determines the results of the different meeting engagements.

45. In Janus, a central server runs the battle. The player stations need only be "dumb" terminals. The server sends out graphical commands to which the player stations respond.

46. To set up a scenario, platforms are allocated to specified player stations. The human player then deploys his forces and assigns initial orders. Once the scenario begins, the players are required to make all the tactical decisions that a commander would make on a real battlefield and the computer determines visibility of friendly and enemy

forces, direct and indirect fire hit and kill probabilities, *etc.* Once the scenario is completed, post-game analysis is possible using data files generated during the simulation.

ModSAF

47. The Modular Semi-Automated Forces model (ModSAF) [6] is a research test-bed, not a production system. Its purpose is to inject and control platform level forces in a virtual simulation. One of the aims of ModSAF is to determine the feasibility of using CGF in the Combined Arms Tactical Trainer (CATT) currently under development. This is a large distributed system that includes actual tank and Armoured Personnel Carrier (APC) simulators in which humans can play out a scenario on a virtual battlefield with other players that may either be CGF or played by other humans. The CGF could be used to generate realistic enemy or flanking forces and their behaviour should not be obviously distinguishable from a HITL simulator of that platform.

48. ModSAF is a real-time constructive simulation with weapon characteristics, platform data, *etc.* It also includes the ability to implement platform-level behaviours, reactions, Standard Operation Procedures (SOPs), and drills. For instance a group of four tanks will act and move as a troop, not four individual co-located tanks. Troops can then be brought together to act as a squadron and they will move and react realistically as an actual squadron would. Platoon / troop level behaviours and drills have been implemented fairly successfully, but the company / squadron level behaviours are more limited. These behaviours remain rule based, but the goal is to use more advanced artificial intelligence (AI) techniques and programming languages based on theories of human decision making.

49. An example of these behaviours was demonstrated. It began with four tanks in an assembly area. When left alone, they would automatically scan their turrets looking for enemy targets. When the troop was given the order to move, they found the nearby road and aligned themselves in a column. The lead tank had its turret facing straight ahead, the next looking in one flanking direction and the third in the other. When the lead vehicle was shot and destroyed by enemy fire, the other tanks were able to move around it and continue along their path without human intervention. While only relatively simple drills or SOPs have been implemented, progress is being made to develop more intricate models.

Simbat

50. Simbat is a stochastic, low-level simulation of military engagements which can be run in both interactive and batch modes [7]. A Simbat scenario consists of a movement network that defines the characteristics of the terrain and a set of forces. Units are allowed to move between significant terrain points, or nodes, which are connected together by arcs along which the actual movement takes place. A force represents a command hierarchy playing a part in the scenario and consists of a set of units and some rules for how those units should behave.

51. Units will only move along paths prescribed for them during the preparation of the scenario. They will follow their routes until the end, at which point it will stop. Units may however make a decision to halt or retreat at some point depending on the tactical situation. Vehicle breakdowns occur randomly according to defined mean distances between failures, but repairs are not modelled. Personnel effectiveness is also reduced at specified rates to reflect fatigue, hunger, thirst, *etc*.

52. As the units move around the scenario, they make regular assessments about what they can see. These sightings involve a probabilistic line of sight check, followed by a randomised comparison against sighting data lookup tables based on factors including available types of observers, terrain, time of day and the motion of the observers and the target.

53. Direct fire occurs through mini-battles in which visible targets are prioritised and attacked, causing casualties of both personnel and equipment. A number of factors are considered, including the nationality of the forces, effectiveness, *etc*. Indirect fire is also provided through a simple model in which assets are controlled by a command unit for the force.

54. Command decisions made by the units include those for controlling movement but may also include orders to stand down or build defences based on the unit state and the perceived combat power ratio.

Clarion

55. The Clarion model [8] is a higher level, two-sided deterministic simulation of land-air combat. Clarion was designed to be a broad-based combat simulation driven by a comprehensive command and control (C2) system. This change in emphasis away from

traditional attrition based models is intended to make CLARION a more flexible and hence, useful tool.

56. All military forces are represented by two types of entities: C2 types and combat entities. The C2 entities provide the communication and command and control and the combat entities conduct the missions assigned to them by C2 entities and interact with each other through their sensors and weapons. These missions may be to

a. Secure. Manoeuvre to a position of advantage and destroy the enemy at a given location;

b. Defend. Prevent the enemy from moving through an area, inflicting maximum casualties;

c. Fix. Move to an area and deny the enemy freedom of action in that area;

d. Move To. Move to a specified area;

e. Search. Search a mission area for any enemy entities;

f. Reserve. Stand in the specified reserve area, ready to be given another mission; or

g. Support. Provide fire support to another commander or entity.

57. The primary use of Clarion is to run large numbers of cases without user interaction in minimum time. It uses data generated by more detailed models, such as Janus to provide results for high level studies. The forces are represented as aggregated entities, typically at brigade level for manoeuvre forces.

III. RECOMMENDATIONS

58. The WGCMC represents an excellent opportunity for both new defence scientists and those being posted to either the Land Forces Operational Research Team (LFORT) or the Research War Gaming Team (RWGT) for the first time. The breadth and depth of material presented on the WGCMC have provided an excellent framework of classical combat modelling techniques and practical experience with modern research and training tools. The scope of the material covered on the course is however not deep enough to justify sending a more experienced analyst who may already have an understanding of

the Lanchester equations or combat models. This course is much better suited to those with only a limited knowledge of these techniques. Junior operational analysts should be encouraged to attend the WGCMC.

59. As a new member of the defence science community, the author especially enjoyed the opportunity to meet other young operational researchers and to gain an appreciation for some of the projects they have previously worked on or are currently facing. Also, since part of the career progression of a defence scientist involves gaining a greater appreciation of the military, this course also provides an opportunity to speak with UK officer cadets and to observe some of the workings of a foreign armed force. This author was also fortunate enough to make a contact at British Aerospace that may be able to provide some direct insights into on-going projects to the Strategic Planning Operational Research Team (SPORT). This possibility is presently being explored.

60. Table II outlines some recommended readings on the topics war gaming and combat modelling. Those that are available in the ORD Library are indicated with a "✓" and the remainder can be obtained through interlibrary loan.

61. Additional information on this and other courses run by AMOR can be found on their web site at http://www.rmcs.cranfield.ac.uk/departments/dois/amorg.

TABLE II
RECOMMENDED READING

Book	ORD Library
• Allen, T.B., *"War Games"*, Heinemann, 1987.	✓
• Ancker Jr., C.J., *"One-On-One Stochastic Duels"*, Military Applications Section, INFORMS, 1982.	✓
• Battilega, J.A. and J.K. Grange (Editors), *"The Military Applications of Modelling"*, Air Force Institute of Technology Press.	✗
• Bracken, J., Kress, M and Rosenthal, R.E. (Editors), *"Warfare Modelling"*, Military Operations Research Society.	✗
• Davis, P.K., *"The Base of Sand Problem"*, RAND papers, 1991.	✗
• Dockery, J.T. and Woodcock, A.E.R. (Editors), *"The Military Landscape: Mathematical Models of Combat"*, Woodhead Publishing, 1993	✗
• Dupuy, T.N., *"Numbers, Predictions and War"*, Hero Books, 1985	✗
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8. *"Clarion Release 2 User Guide"*, CORDA, 1997.

ADMINISTRATIVE DETAILS

Location

1. The WGCMC is run at Cranfield University's Shrivenham campus at the RMCS. Shrivenham is in the Wiltshire province of the United Kingdom. Approximately a fifteen-minute drive from the campus is nearest large city, Swindon, which is a two-hour drive directly East of London. Figures (A-1) through (A-3) show the area of England where RMCS is located in increasing detail.

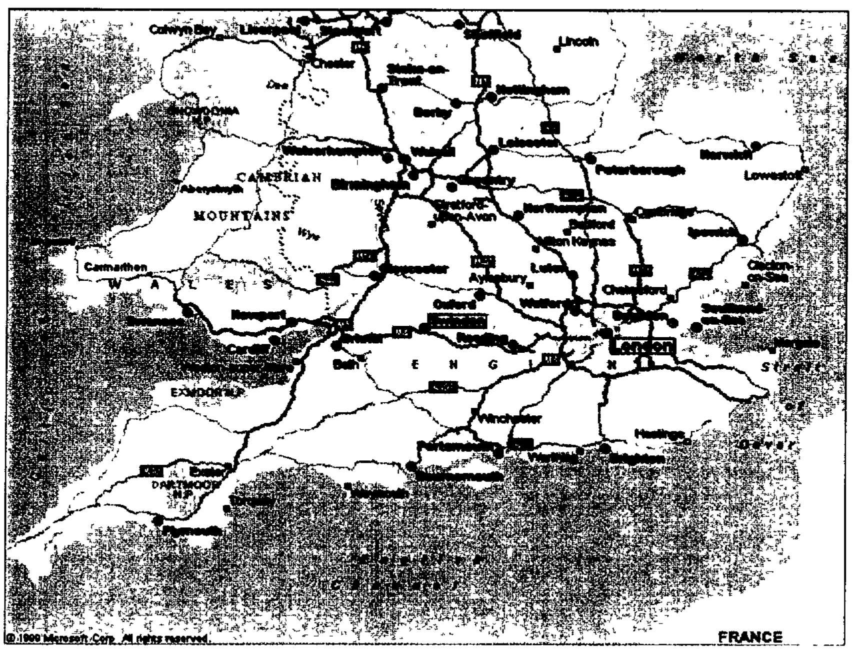

Figure A-1 – Map of Southern United Kingdom.

2. Sleeping quarters are often available on the campus and there are limited numbers Bed and Breakfasts nearby. This author was assigned sleeping quarters in Roberts Hall, the Officer's Mess on campus. This is located a fifteen minute walk from Marlborough Hall where the majority of the classes were held. All meals were also served in Roberts Hall.

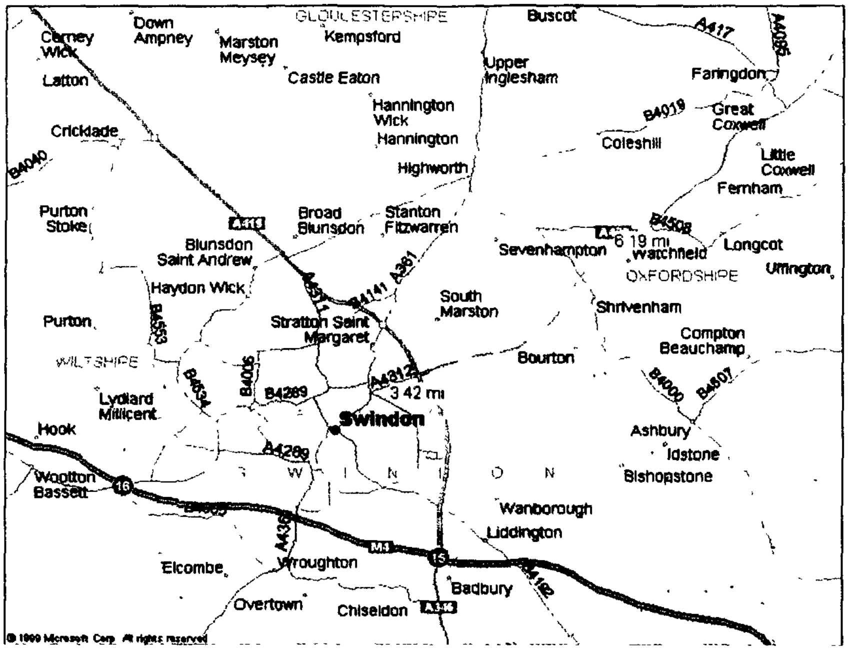

Figure 2 – The Swindon area of the Wiltshire province.

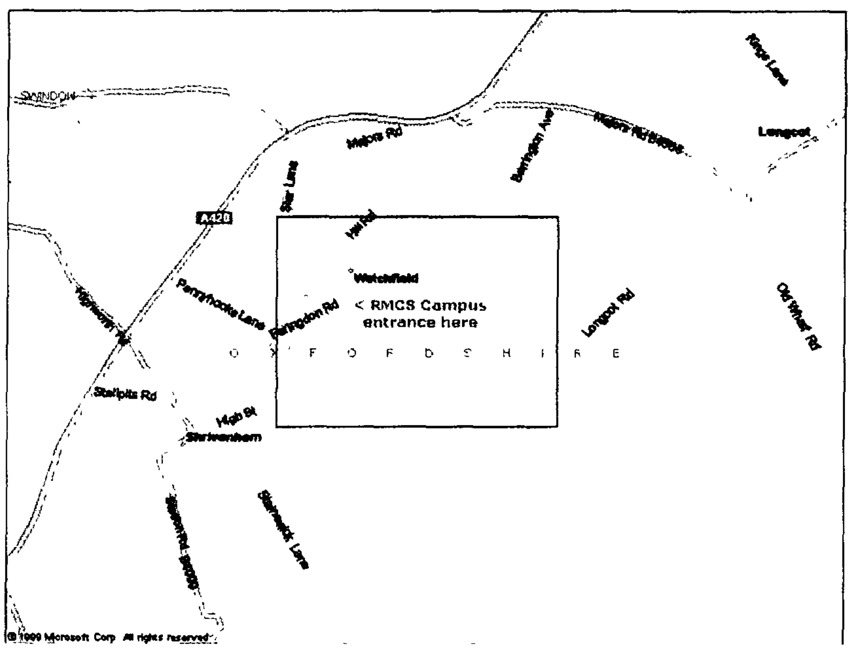

Figure 3 – The precise location of RMCS in Shrivenham and Watchfield.

Costs

3. The cost of the WGCMC was £1000, which did not include accommodations. At the present exchange rate of 2.26, this corresponds to $2260 CDN. This covers all instruction and course materials, as well as computer time in the Simulation and Synthetic Environment Laboratory. The SSEL was open for a limited amount of time outside of normal class hours for students that wanted extra instruction, but the SSEL is also utilised by other RMCS students, so resources were not always available.

4. Being a member of the Canadian Primary Reserve force, this author was entitled to a cheaper rate of accommodations at Roberts Hall than would normally be available to ORD civilian personnel. Table A-I shows a breakdown of both the entitled and non-entitled rates.

TABLE A-I
MESSING AND ACCOMMODATION COSTS IN ROBERTS HALL FOR CANADIAN MILITARY PERSONNEL AND CIVILIANS

	Entitled (Military)		**Non-Entitled (Civilian)**	
Item	**Pound Sterling (£)**	**Canadian Dollars ($)**	**Pound Sterling (£)**	**Canadian Dollars ($)**
Messing	11.80	26.67	122.75	277.42
Accommodation	9.20	20.79	30.70	69.38
Extra messing	4.90	11.07	4.90	11.07
Military Subscriptions	1.25	2.83	0.00	0.00
Civilian Subscriptions	0.00	0.00	4.70	10.62
Totals	**27.15**	**61.36**	**163.05**	**368.49**

5. While a significant saving can be realised by a member of the CF, staying on campus is very modestly priced for civilian personnel. While exact amounts are not known, a room for a single person staying in a bed and breakfast in that part of England was typically between £30 and £50 or $67.80 to $113.00 Canadian per night. Over six nights and including the £50 daily meal allowance, the cost would be between $1084.80 and $1356. Clearly, if rooms are available on campus, future ORD personnel taking any courses at RMCS should arrange to stay there.

6. Other costs for this course are given in Table A-II.

TABLE A-II
OTHER COSTS TO ATTEND THE WAR
GAMING AND COMBAT MODELLING COURSE

Item	Cost Canadian Dollars ($)
Return Flight	$978.29
Daily Incidentals and Claimed Meals	$336.87
Passport	$60.00
Passport Photo	$11.49
Hotels (2 nights)	$207.81
Taxis, Buses, Car Rental[1]	$544.22

7. The total cost incurred by the ORD was $4460.04.

[1] Originally, a rental car was not authorised for this trip. However, since the author was travelling with a companion, it was decided that it would be worthwhile for us to rent a car for use in the evenings. DOR(J&L) agreed to cover the £140 it would have cost for a return trip from the airport to RMCS via Brian's Hire taxi service that is recommended by AMOR in the joining instruction for the WGCMC. The total rental cost for an economy car for one week with unlimited mileage was £178.

STUDENT EXERCISES

1. The students of the War Gaming and Combat Modelling Course completed several exercises designed to reinforce the concepts taught each day. This Annex contains the questions that the students were asked to solve and their solutions.

2. The exercises and their solutions are organised in separate Appendices. See Table B-I for a brief description of each exercise.

TABLE B-I
EXERCISES ON THE WAR GAMING
AND COMBAT MODELLING COURSE

Exercise Name	Description
Ominous Reception	Basic concepts of a Monte Carlo simulation of combat
Orbicular Switch	Develop Lanchester theory
Orthodox Approach	Application of stochastic Lanchester Square Law
Oregon Trail	Measures of effectiveness and linear programming

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OPERATIONAL ANALYSIS

EXERCISE OMINOUS RECEPTION

INTRODUCTION

1. Simulation techniques provide a powerful tool for the analysis of military conflicts and such techniques are widely used in R&D establishments and in industry.

AIM

2. The aim of this exercise is to demonstrate some of the basic concepts involved in a 'Monte Carlo' simulation of a simple combat situation.

THE ENGAGEMENT

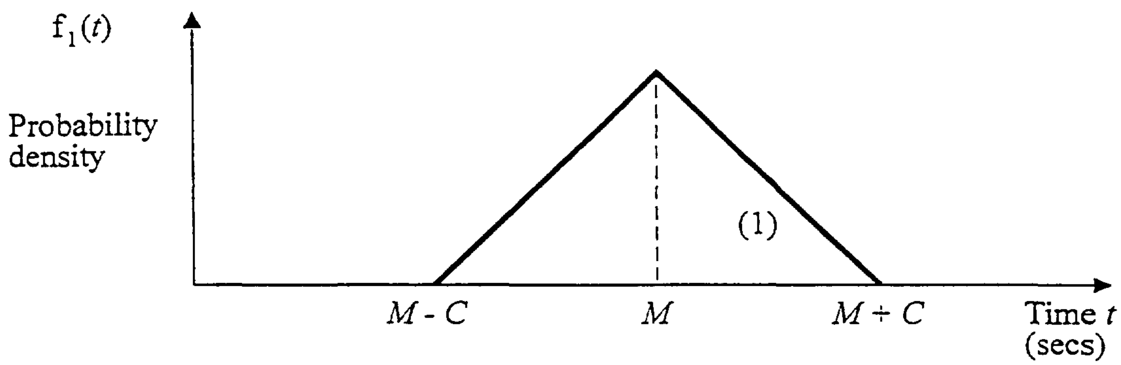
3. In this exercise you will carry out a 'Monte Carlo' simulation to determine the likely outcome of the engagement described below, in which one of the 2 sides is fought to annihilation.

   (a) Force X has one tank and 2 vehicle mounted ATGW. Force Y has 4 tanks.

   (b) The single shot hit/kill probabilities are the probabilities that a single shot hits and kills its target and are as follows:

Tank firing at tank	:	0.4
Tank firing at ATGW	:	0.3
ATGW firing at tank	:	0.8

(c) When an ATGW engages a new tank or fires again at the same tank there is a certain time interval before the missile strikes and either hits or misses its target. Also when a tank engages a new target there is a certain time interval before the shell strikes and either hits or misses its target. These time intervals are assumed to have a triangular probability density function as follows:



Note:

(1) The theory of probability requires that the area under the triangle must be unity.

(d) The mathematical expression describing this probability density function is:

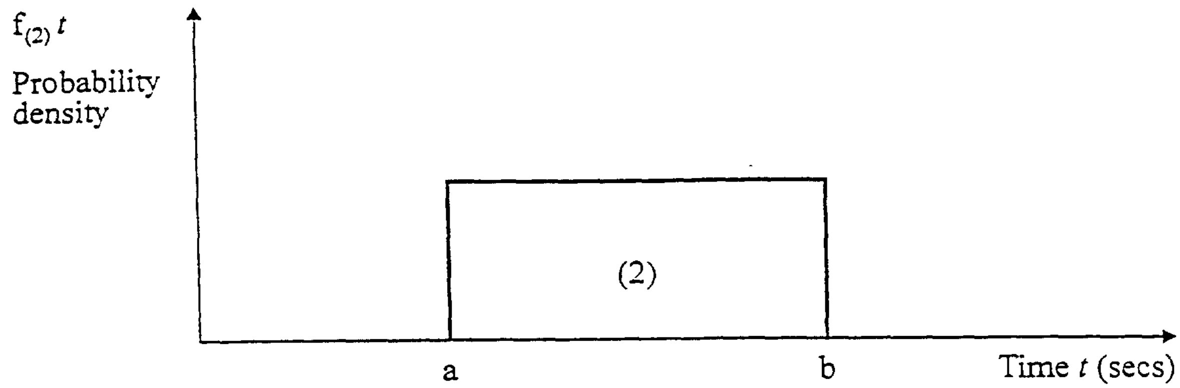
$$f_1(t) = \begin{cases} \dfrac{1}{C} + \dfrac{(t-M)}{C^2} & M - C < t < M \\[2ex] \dfrac{1}{C} + \dfrac{(M-t)}{C^2} & M < t < M + C \end{cases}$$

where for ATGW : $M = 5, \ C = 3$

for Tanks : $M = 4, \ C = 2$

M is the mean time to engagement and C is a constant to allow for the spread of actual engagement times from the mean. Graphs of the related cumulative distribution functions are given at Annex A.

(e) If a tank fails to defeat its target, it fires again against the same target and the time intervals between firings have a rectangular distribution with probability density function as follows:



Note:

(2) The theory of probability demands that the area under the rectangle must be unity.

(f) The mathematical expression describing this probability density function is:

$$f_2(t) = \frac{1}{b-a} \qquad a < t < b$$

where for X tanks : $a = 1, \; b = 3$

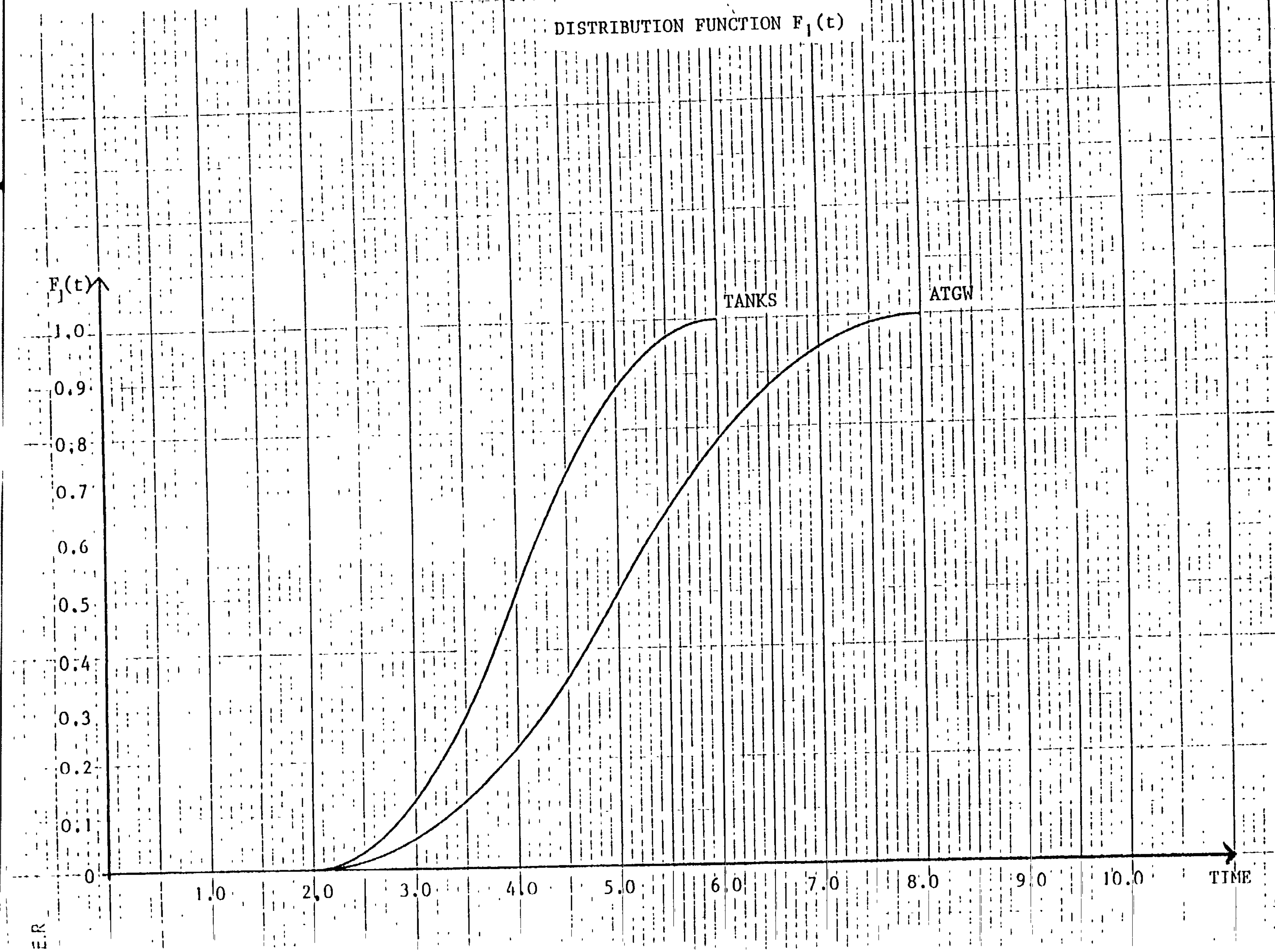
for Y tanks : $a = 2, \; b = 4$

a is the shortest time a re-engagement will take and b is the longest time.

(g) Graphs of the related cumulative distribution functions are given at Annex B.

(h) Each force attempts to spread its fire as evenly as possible over the opposing force. If this results in a choice of targets a random selection is made, except that in these cases tanks are always engaged in preference to ATGW vehicles.

Annex A Distribution Function $f_1(t)$
Annex B Distribution Function $f_2(t)$
Annex C Event and Status Register



DISTRIBUTION FUNCTION $F_1(t)$
$F_j(t)$
TANKS
ATGW
TIME
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
9.0
10.0

EXERCISE OMINOUS RECEPTION

EXPLANATION OF ALGORITHM

1. This exercise is an example of a simulation technique known as event-stepping. That is to say, the situation being studied is broken down into a number of events (in this case, firing one tank gun or ATGW at a target). The events are analysed in the order in which they occur. The analysis may include all or any of the following:

   a. The use of random number generators.

   b. The removal of some subsequent event or events.

   c. The generation of a further event or events (and their times).

2. The analysis in this case is summarised in the flow diagram at Annex E. A fuller explanation is as follows:

   a. Using the priority rules given in the descriptions of the exercise, pick a target for each weapon. If the rules fail to define a unique target then use the random number calculator to choose between those targets which are equally likely. For example, suppose we are to pick a target for the X tank, and the targets for the ATGWs have not yet been chosen. Label the Y tanks 1-4 and generate a random number. If this number lies between the limiting values of 0 and .25, the target is tank no 1, if between .25 and .5, it is tank no 2 and so on. This technique, with appropriate limiting values, will be used whenever the rules require that a target is to be chosen for any weapon.

   b Again using the random number generator, choose a firing time for each weapon. This is done as follows. Select the cumulative distribution curve corresponding to the appropriate distribution of times to fire from Annex B or C. Generate a random number. Using the curve, find the number on the horizontal axis corresponding with the generated random number on the vertical axis. This is the required time, and the times so chosen will have the appropriate distribution

   c. All the initial events will now have been generated. Each event consists of a firing weapon, its target (randomly chosen within the priority constraints) and its time to fire (also randomly chosen from the appropriate distribution).

d. Analyse the outcome of the event with the lowest time First decide, using the random number generator, if the weapon has killed its target. If p is the probability of a kill, generate a random number, and if this is less than p, the target has been killed, otherwise, the target has not been killed.

e. If the target has not been killed, the weapon will fire again at the same target. Select a firing time from the appropriate distribution. A new event is generated in which the firing weapon and target are the same as the old event, but the firing time is the time of the old event plus the firing time just generated. Now go back to d.

f. If the target has been killed, the situation is more complex. A number of events that might have taken place will not now do so, and have to be removed and possibly replaced. The event that consists of the target (now destroyed) firing at its target will not now take place, and must be removed. Any event which has as its target the now destroyed target will have to be removed. If there are no targets remaining, the simulation is now finished, and the results recorded; otherwise new targets and firing times will be picked using the techniques described in a-e above.

3. **Conclusion**: The simulation must be repeated many times in order to determine whether either side has an advantage, and to be able to quantify that advantage.

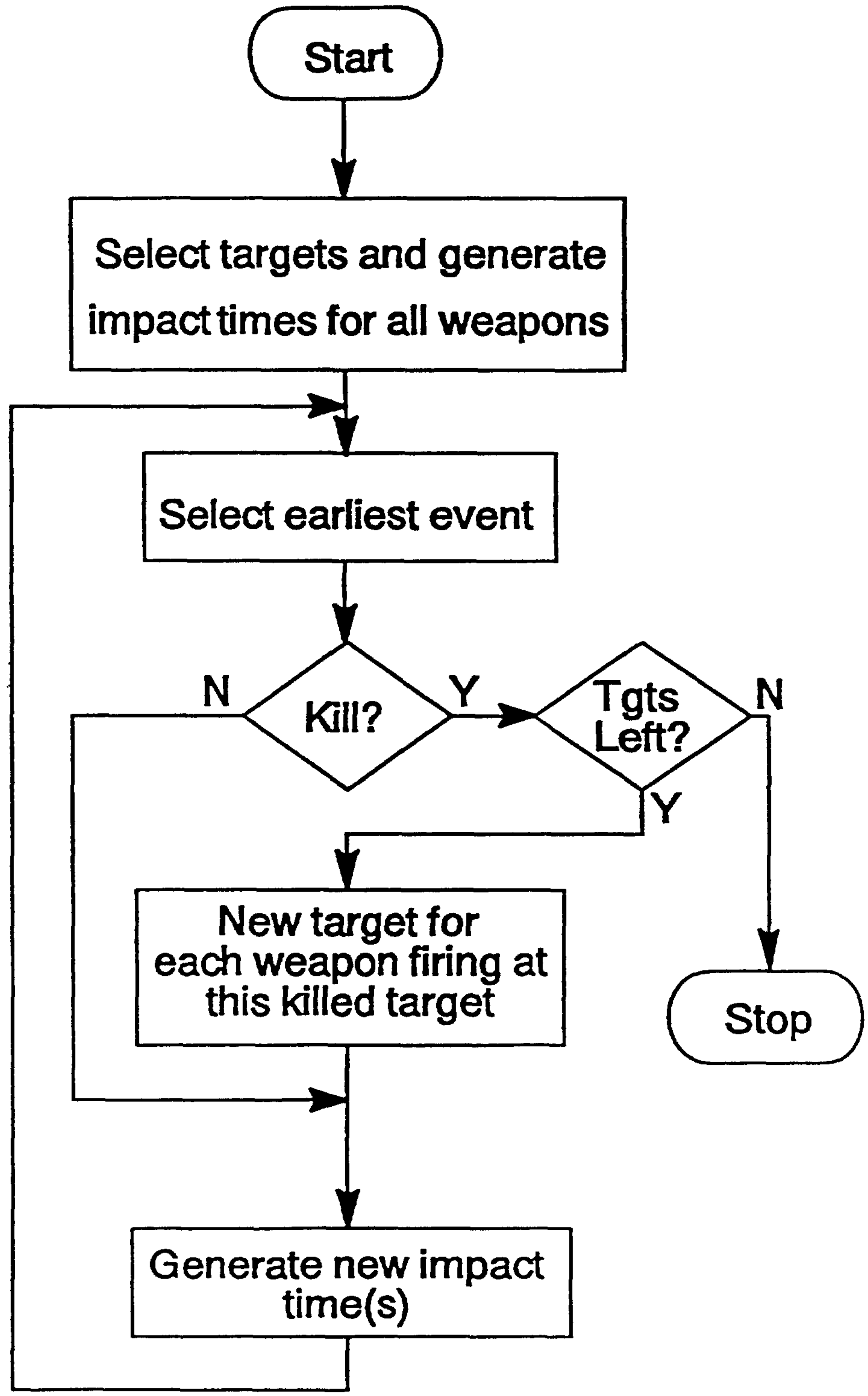
4. The following further points must be noted:

a. This simulation has been simplified in order to satisfy the requirements of the classroom exercise, by omitting such details as flight times of missiles, defenders firing first from prepared positions etc. Such factors can easily be included in simulations used in the study of combat.

b. Carrying out simulations manually is time-consuming and tedious. However, using the power of modern computers, many simulations can be carried out in a relatively short time.

c. If too much detail is demanded of simulation in a search for greater realism, the programme can become so complex that no one person is able to comprehend the effects of all the interaction played, and the requirement for computing power can become enormous.

EXERCISE OMINOUS RECEPTION – FLOW DIAGRAM



EX OMINOUS RECEPTION

RESULTS

						Average
Replications	100	100	100	100	100	100
X wins	48	47	47	41	38	44.2
Y wins	52	53	53	59	62	55.8
Battle time	11.66	11.52	10.93	10.10	10.48	10.94
No. of shots	10.89	10.57	10.46	9.38	9.64	10.19
X Tanks	0.35	0.40	0.43	0.37	0.39	0.39
X GW	1.52	1.49	1.53	1.56	1.47	1.51
Y Tanks	2.48	2.11	2.47	2.53	2.63	2.44

EX OMINOUS RECEPTION

RESULTS

						Average
Replications	1000	1000	1000	1000	1000	1000
X wins	443	473	426	482	461	457
Y wins	557	527	574	518	539	543
Battle time	11.05	10.96	10.99	11.27	11.16	11.09
No. of shots	10.41	10.35	10.45	10.68	10.61	10.50
X Tanks	0.39	0.44	0.45	0.40	0.41	0.42
X GW	1.53	1.52	1.47	1.51	1.50	1.51
Y Tanks	2.48	2.51	2.48	2.48	2.47	2.48

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OPERATIONAL ANALYSIS

EXERCISE ORBICULAR SWITCH

INTRODUCTION

1. It is often stated that the killing rates that occur in Lanchester's Equations may be determined by multiplying the chance that a round hits and kills the enemy by the rate of fire achieved - or more precisely by the average number of rounds that will be fired in unit time.

2. This exercise is concerned with obtaining a mathematical expression for this latter quantity that takes into account not only the cyclic rate of fire of the weapon - that is, the maximum rate of aimed fire that can be achieved against a particular target during an engagement allowing for the time needed for loading, laying and firing - but also of the time needed to switch from target to target. In the case of a tank (which is to be considered in this exercise) switching time depends for example on the angle through which the turret has to be traversed from one target to another, on the rate of traverse and on the time taken to make a fine lay on the new target.

ASSUMPTIONS AND NOTATION

3. Define

S (seconds)	=	average time required to switch from one target to another; more precisely, it is the time between the last round fired at one target (the killing round) and the first round fired at the next.
T (seconds)	=	average time between rounds fired at the same target (Cyclic rate of fire = $60/T$ rounds per minute).
$P = (1 - Q)$	=	probability that a round fired at an enemy tank will kill it.

4. It is to be assumed

   (a) That a tank will continue to engage a target until it kills it and will then be able to switch immediately to engage a new target. This is the usual Lanchester assumption.

   (b) That P is constant and independent of the number of rounds previously fired at a target.

QUESTIONS

5. (A) What is the frequency with which a tank will have to fire 1, 2, 3, ...n.. rounds against an enemy tank before killing it?

   (B) What is the average number of rounds that has to be fired against an enemy tank to kill it?

   (C) If in a particular engagement 'n' rounds have to be fired against an enemy tank before it is killed, what is the time taken from the last round fired at the previous target?

   (D) What is the number of rounds fired in unit time, averaged over a large number of individual engagements (in which 1, 2, 3, ... rounds are fired with the frequency determined in (a))?

   (E) Plot a graph based on the expression just derived which shows how the average number of rounds fired in unit time varies with P. Comment on it. (Take $S = 9$ sec, $T = 6$ sec.)

   (F) If for an enemy tank $S = 7$ sec, $T = 5$ sec and $P = 0.7$ and for our own tank $S = 9$ sec and $T = 6$ sec, what value of P must we achieve to have an effectiveness of 1.0 relative to the enemy?

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OPERATIONAL ANALYSIS

EXERCISE ORBICULAR SWITCH

SOLUTION

1. QUESTION A

What is the frequency with which a tank will have to fire 1, 2, 3, ...n, ... rounds against an enemy tank before killing it?

The probability that n rounds have to be fired to kill the enemy tank
= (probability that $(n-1)$ rounds fail to kill it) × (probability that nth round does)

$$= (1-P)^{n-1}.P = Q^{n-1}.P = f(n) \tag{1}$$

Thus the frequency with which a tank will have to fire 1, 2, 3, ... rounds is:

P, QP, Q^2P, ...

2. QUESTION B

What is the average number of rounds that has to be fired against a tank in order to kill it?

Average number of rounds fired,

$$N = 1.f(1) + 2.f(2) + 3.f(3) + \ldots \tag{2}$$

Thus from Equation (1)

$$\begin{aligned} N &= P + 2QP + 3Q^2P + \ldots \\ &= P(1 + 2Q + 3Q^2 + \ldots) \end{aligned}$$

Multiply both sides by $(1 - Q)$

$$\begin{aligned} N(1-Q) &= P(1 - Q + 2Q - 2Q^2 + 3Q^2 - 3Q^3 \ldots) \\ &= P(1 + Q + Q^2 + \ldots) \end{aligned}$$

Multiply both sides by $(1 - Q)$ again

$$\begin{aligned} N(1-Q)^2 &= P(1 - Q + Q - Q^2 + Q^2 - Q^3 + \ldots) \\ &= P \end{aligned}$$

$$\text{Thus} \qquad N = \frac{P}{(1-Q)^2} = \frac{P}{P^2} = \frac{1}{P} \tag{3}$$

For example, if $P = 0.5$, on an average 2 rounds will have to be fired against a target to kill it.

3. QUESTION C

If in a particular engagement 'n' rounds have to be fired against an enemy tank before it is killed, what is the time taken from the last round fired at the previous target?

The time from the last round fired at the previous target to the nth round being fired is equal to the switching time plus $(n - 1)$ intervals between rounds.

Thus

$$t_n = S + (n-1)T \tag{4}$$

4. QUESTION D

What is the number of rounds fired in unit time. averaged over a large number of individual engagements (in which 1, 2, 3, ... rounds are fired with the frequency determined in A)?

The tank will on average fire

$$1.\ f(1) + 2.\ f(2) + 3.\ f(3) + \ldots \tag{5}$$

rounds in a time

$$t \cdot f(1) + t_2 \cdot f(2) + t_3 \cdot f(3) \ldots \text{seconds} \tag{6}$$

The first of these expressions is the same as that for N (Equation (2)) and hence equals

$$\frac{1}{P} \tag{7}$$

Using Equations (1) and (4) the second expression equals

$$\begin{aligned} & SP + (S+T)QP + (S+2T)Q^2P + \ldots \\ = \; & SP(1 + Q + Q^2 \ldots) \\ & \qquad + QPT(1 + 2Q + 3Q^2 + \ldots) \\ = \; & \frac{SP}{(1-Q)} + \frac{QPT}{(1-Q)^2} \\ = \; & \frac{SP}{P} + \frac{(1-P)PT}{P^2} \\ = \; & (S + \frac{(1-P)}{P}T) \end{aligned} \tag{8}$$

Thus in unit time the tank will on an average fire a number of rounds, N', equal to

$$\frac{1}{P} \div (S + \frac{(1-P)}{P}\, T)$$

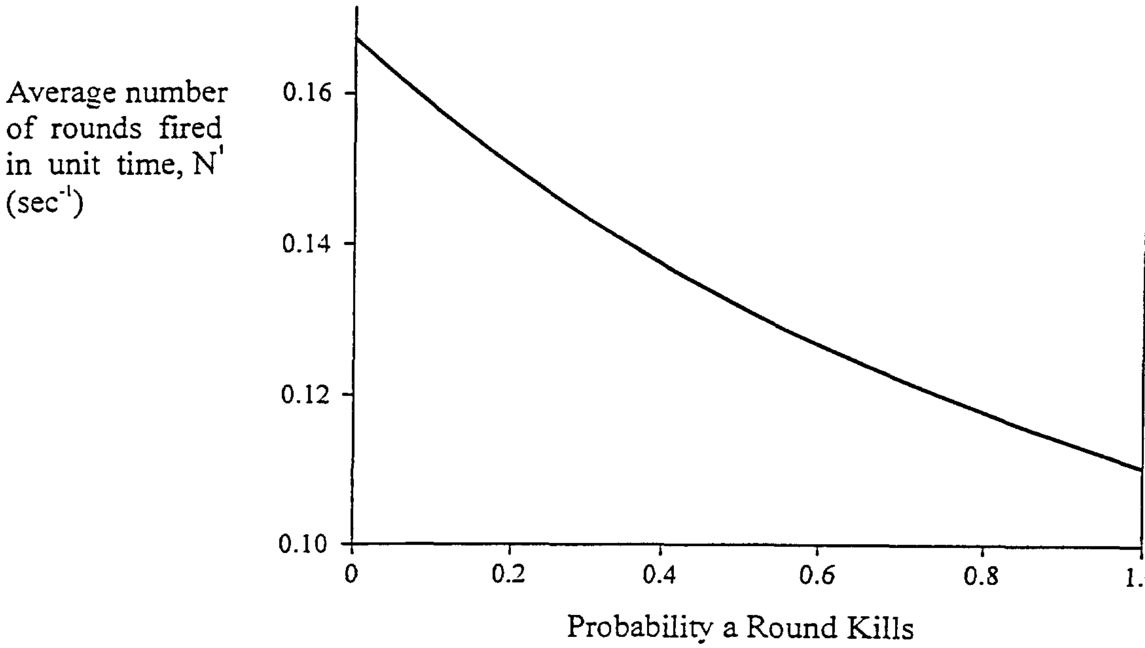
That is

$$N' = \frac{1}{PS + (1-P)T} \tag{9}$$

5. QUESTION E

Plot a graph based on the expression just derived which shows how the average number of rounds fired in unit time varies with P. Comment on it. (Take $S = 9$ sec, $T = 6$ sec).

P	0	0.2	0.4	0.6	0.8	1.0
$N' = \frac{1}{9P+6(1-P)}$	0.167	0.152	0.139	0.128	0.119	0.111



Comments:

(a) When $P = 0$, $N' = \frac{1}{T}$ (from Equation (9)).
This is because when there is zero chance of kill, the tank fires an infinite number of rounds at the same target and thus the number of rounds fired in unit time is equal to the cyclic rate of fire.

(b) When $P = 1$, $N' = \frac{1}{S}$
In this case, every round kills. Thus the tank spends its time switching from target to target between rounds and it is the speed of switching only that determines the number of rounds fired in unit time.

6. QUESTION F

If for an enemy tank $S_R = 7$ sec, $T_R = 5$ sec, and $P_P = 0.7$, and for our own tank $S_B = 9$ sec and $T_B = 6$ sec, what value of P must we achieve to have an effectiveness of 1.0 relative to the enemy?

It follows directly from Lanchester's Equations that the effectiveness of our tank will be equal to that of the enemy if the killing rate of our tank, k_B, equals the killing rate of his, k_R.

Now $$k_R = P_R \times N'_R$$

$$= \frac{P_R}{P_R S_R + (1 - P_R) T_R} = 0.109$$

Similarly $$k_B = \frac{P_B}{9P_B + (1 - P_B).6} = \frac{P_B}{3P_B + 6}$$

Thus $$\frac{P_B}{3P_B + 6} = 0.109$$

or $$P_B = 97.2\%$$

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OPERATIONAL ANALYSIS

EXERCISE ORTHODOX APPROACH

INTRODUCTION

1. The deterministic Lanchester Equations are mathematical models that have found many applications in the analysis of combat. The Lanchester 'Square-Law' Equation particularly is often used in deterministic models of battle between direct-fire weapons.

2. If probabilistic outcomes of battle are required stochastic formulation of the Lanchester Equations must be used in the analysis of the attrition process.

3. This exercise is concerned with the application of a stochastic formulation of the Lanchester 'Square-Law' Equation to the analysis of a tank vs tank direct-fire battle.

4. Reference should be made to precis OA 824 'The Stochastic Lanchester Equations' and to OA 1543 'Exercise Orbicular Switch' as necessary.

THE TANK vs TANK BATTLE

5. Two Blue tanks are to engage three Red tanks in a direct-fire battle. The characteristics of the tanks in the battle and the terrain over which the battle is to be fought give the following input values required for analysis.

	Blue Tank	Red Tank
Average time required to switch from one enemy target to another (units of time)	3	6
Average time between rounds fired at the same enemy target (units of time)	2	3
Probability that a round fired at an enemy tank will kill it.	0.5	0.4

6. The usual Lanchester assumptions hold for the battle:

   (a) The total firepower of each side can be brought to bear on the other. A uniform spread of firepower is maintained by each side, over the opposition, throughout the battle.

   (b) Each tank on each side kills tanks on the other at a constant rate until either the battle ceases or the tank itself is killed.

PROBLEMS FOR BATTLE ANALYSIS

7. The following problems should be considered:

Problem 1 - Calculate the kill rates of the individual Blue and Red tanks.

Problem 2 - Construct a tree diagram to show the possible intermediate and final battle outcomes.

Problem 3

(a) Construct appropriate matrices giving the transition probabilities from one battle state to the next.

(b) Use the tree diagram and the transition probabilities to calculate the probability that the Blue force will defeat Red (that is, destroy it).

(c) If Blue defeats Red determine the expected number of Blue tanks that survive.

(d) If Red defeats Blue determine the expected number of Red tanks that survive.

Problem 4

(a) Construct a matrix giving the expected transition times from one battle state to the next.

(b) Use the tree diagram and the transition times to calculate the expected duration of the battle (ie the expected time to the defeat of Red or Blue).

(c) If Blue defeats Red determine the expected duration of the conflict.

(d) If Red defeats Blue determine the expected duration of the conflict.

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OPERATIONAL ANALYSIS

EXERCISE ORTHODOX APPROACH

SOLUTION FOR PROBLEM 1

1. Notation

Number of Blue tanks at start of battle	B
Number of Red tanks at start of battle	R
Number of Blue tanks at time t from start of battle	b
Number of Red tanks at time t from start of battle	r
Kill-rate of a Blue tank	β
Kill-rate of a Red tank	ρ

2. Solution

Using the notation adopted in OA 1543 Exercise Orbicular Switch let:

S = Average time required to switch from one target to another.

T = Average time between rounds fired at the same target.

P = Probability that a round fired at an enemy tank will kill.

3. From OA 1543 the kill rate of a rank is given by

$$\frac{P}{PS + (1-P)T}$$

Substituting the appropriate values into the expression we have:

For the Blue force $\beta = \dfrac{0.5}{(0.5 \times 3) + (0.5 \times 2)} = 0.200$

For the Red force $\rho = \dfrac{0.4}{(0.4 \times 6) + (0.6 \times 3)} = 0.0952$

4. **Note:** Although the kill rate of the individual Red tanks is less than half that of the Blue tanks, if the two tank **forces** are considered, an interesting result is obtained. The deterministic Lanchester 'Square-Law' Equation may be written as:

$$\beta(B^2 - b^2) = \rho(R^2 - r^2)$$

For parity between the two forces the following condition must hold:

$$\frac{\beta}{\rho} = \frac{R^2}{B^2}$$

If $R = 3$ and $B = 2$ then for parity between these two forces we require

$$\frac{\beta}{\rho} = \left(\frac{3}{2}\right)^2 = 2.25$$

The actual ratio for $\dfrac{\beta}{\rho} = \dfrac{0.20}{0.0952} = 2.10$

Hence the deterministic equation implies that the two forces are close to parity, even though the individual kill rates are dissimilar.

OPERATIONAL ANALYSIS
EXERCISE ORTHODOX APPROACH

SOLUTION FOR PROBLEM 2

Tree gives (r, b)
Possible battle states.

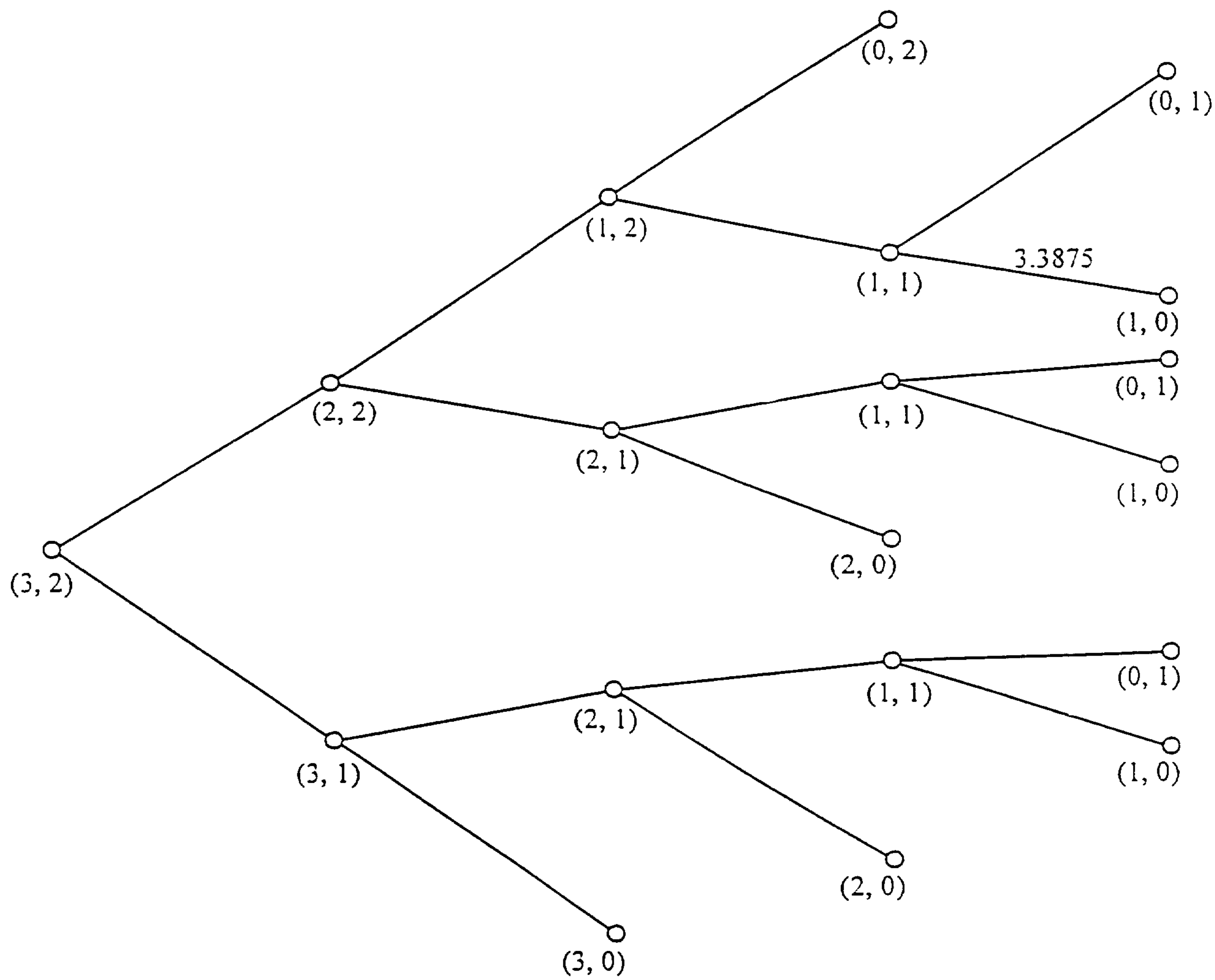

Figure 1

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OPERATIONAL ANALYSIS

EXERCISE ORTHODOX APPROACH

SOLUTION FOR PROBLEM 3(a)

1. Reference should be made to OA 824.

2. To construct matrices giving the transition probabilities from one battle state to the next it is necessary to use the conditional probabilities that the next battle casualty is either a Red tank or a Blue tank.

   Probability that next battle casualty is a red tank (ie kill is by a Blue tank) $= \dfrac{\beta b}{\beta b + \rho r}$.

   Probability that next battle casualty is a Blue tank (ie kill is by a Red tank) $= \dfrac{\rho r}{\beta b + \rho r}$.

   Note that since $\dfrac{\beta b}{\beta b + \rho r} + \dfrac{\rho r}{\beta b + \rho r} = 1$, the next casualty must be either a Red or a Blue tank.

3. The expressions may be used to construct the tables of transition probabilities as follows:

		Red r	
	3	2	1
Blue b 2	.5834	.6775	.8078
Blue b 1	.4119	.5123	.6775

Table gives the probability that if battle state (r, b) exists the **next** battle casualty is a Red tank.

		Red r	
	3	2	1
Blue b 2	.4166	.3225	.1922
Blue b 1	.5881	.4877	.3225

Table gives the probability that if battle state (r, b) exists the **next** battle casualty is a Blue tank.

SOLUTIONS FOR PROBLEMS 3(b), 3(c), 3(d)

4. Transition probabilities from the Tables are marked as appropriate against the tree diagram in Figure II. The probability that any path of the tree will occur can be calculated from the individual probabilities along that path. For example, the probability that path (3, 2); (2, 2); (1, 2); (0, 2) will occur is given by $.5834 \times .6775 \times .8078 \times .3193$. This is then the probability that the battle will end with a Red defeat and with both Blue tanks surviving. Each path through the tree may be similarly evaluated.

5. The following tables give the probabilities of the different battle outcomes:

Number of Red tanks at battle end	Probability of this outcome	Conditional probability given a Red win of the number of surviving Red tanks
0	.4957	0
1	.0839	.1663
2	.1755	.3479
3	.2450	.4857

Number of Blue tanks at battle end	Probability of this outcome	Conditional probability given a Blue win of the number of surviving Blue tanks
0	.5044	0
1	.1764	.3559
2	.3193	.6441

6. Note that the probability of a Red win is 0.5044, the probability of a Blue win 0.4957. The two forces are close to parity, as suggested by the deterministic equation.

7. The conditional probabilities yield the expected number of tanks surviving, given a Red or Blue win.

   Expected number of Red tanks surviving given a Red Win
   $= (1 \times .1663) + (2 \times .3479) + (3 \times .4857) = 2.3192 = 2.3$ (1 d.p.)

   Expected number of Blue tanks surviving given a Blue win
   $= (1 \times .3559) + (2 \times .6441) = 1.6441 = 1.6$ (1 d.p.)

OPERATIONAL ANALYSIS

EXERCISE ORTHODOX APPROACH

SOLUTION FOR PROBLEMS 3(b), 3(c), 3(d) (CONT'D)

Tree gives (r, b)

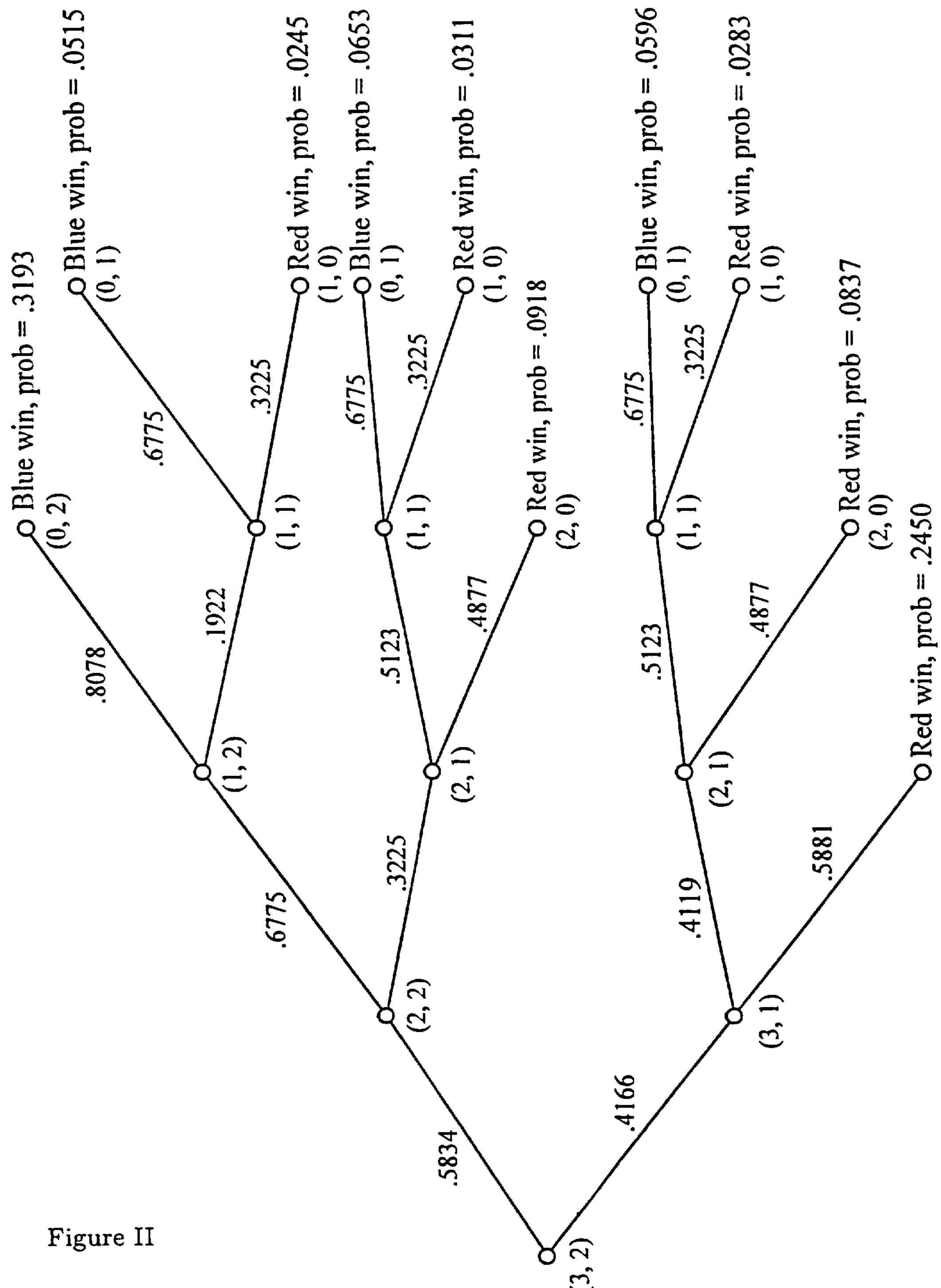

Figure II

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OPERATIONAL ANALYSIS

EXERCISE ORTHODOX APPROACH

SOLUTION FOR PROBLEM 4(a)

1. Reference should be made to OA 824.

2. The expected transition time from one battle state to the next is given by

$$\frac{1}{\beta b + \rho r}$$

3. The expression may be used to construct the following table of expected transition times for the battle.

		Red r		
		3	2	1
Blue b	2	1.4586	1.6938	2.0194
	1	2.0593	2.5615	3.3875

Table gives the mean time for transition from battle state (r, b) to battle state $(r-1, b)$ or $(r, b-1)$.

SOLUTIONS FOR PROBLEMS 4(b), 4(c), 4(d)

4. Mean transition times from the above Table are marked as appropriate against the tree diagram in Figure III. The expected duration of any path of the tree is the sum of the expected transition times along that path. For example, the expected duration of path (3, 2); (2, 2); (1, 2); (0, 2) is given by

$$1.4586 + 1.6938 + 2.0194 = 5.1718$$

All other paths may be similarly enumerated.

5. To calculate the mean duration of the battle it is necessary to consider the probability that a particular path will actually occur. These probabilities are given in Figure II. The expected duration of the battle is a weighted mean of the durations of the possible different paths that may occur during the battle.

6. The expected duration of battle is given by the sum of the following products:

$$\begin{aligned}
5.1718 \times 0.3191 &= 1.6513 \\
8.5593 \times 0.0515 &= 0.4408 \\
8.5593 \times 0.0245 &= 0.2097 \\
9.1014 \times 0.0653 &= 0.5943 \\
9.1014 \times 0.0311 &= 0.2831 \\
5.7139 \times 0.0918 &= 0.5245 \\
9.4669 \times 0.0595 &= 0.5642 \\
9.4669 \times 0.0283 &= 0.2679 \\
6.0794 \times 0.0837 &= 0.5088 \\
3.5179 \times 0.2450 &= \underline{0.8619} \\
& 5.9065
\end{aligned}$$

Expected duration of the battle = 5.9 units of time(1 d.p.).

7. Figure III gives, against each path as appropriate, the conditional probability that the path will occur if either a Blue or Red win results. These conditional probabilities may be used, as in paragraph 6, to determine the expected duration of the battle given either a Blue or Red win. Expected duration of the battle, given a Blue win = 6.6 units of time. Expected duration of the battle, given a Red win = 5.3 units of time.

OPERATIONAL ANALYSIS
EXERCISE ORTHODOX APPROACH

SOLUTIONS FOR PROBLEMS 4(b), 4(c), 4(d) (CONT'D)

Tree gives (r, b)

[ ] Conditional probability of path given a Blue win.

< > Conditional probability of path given a Red win.

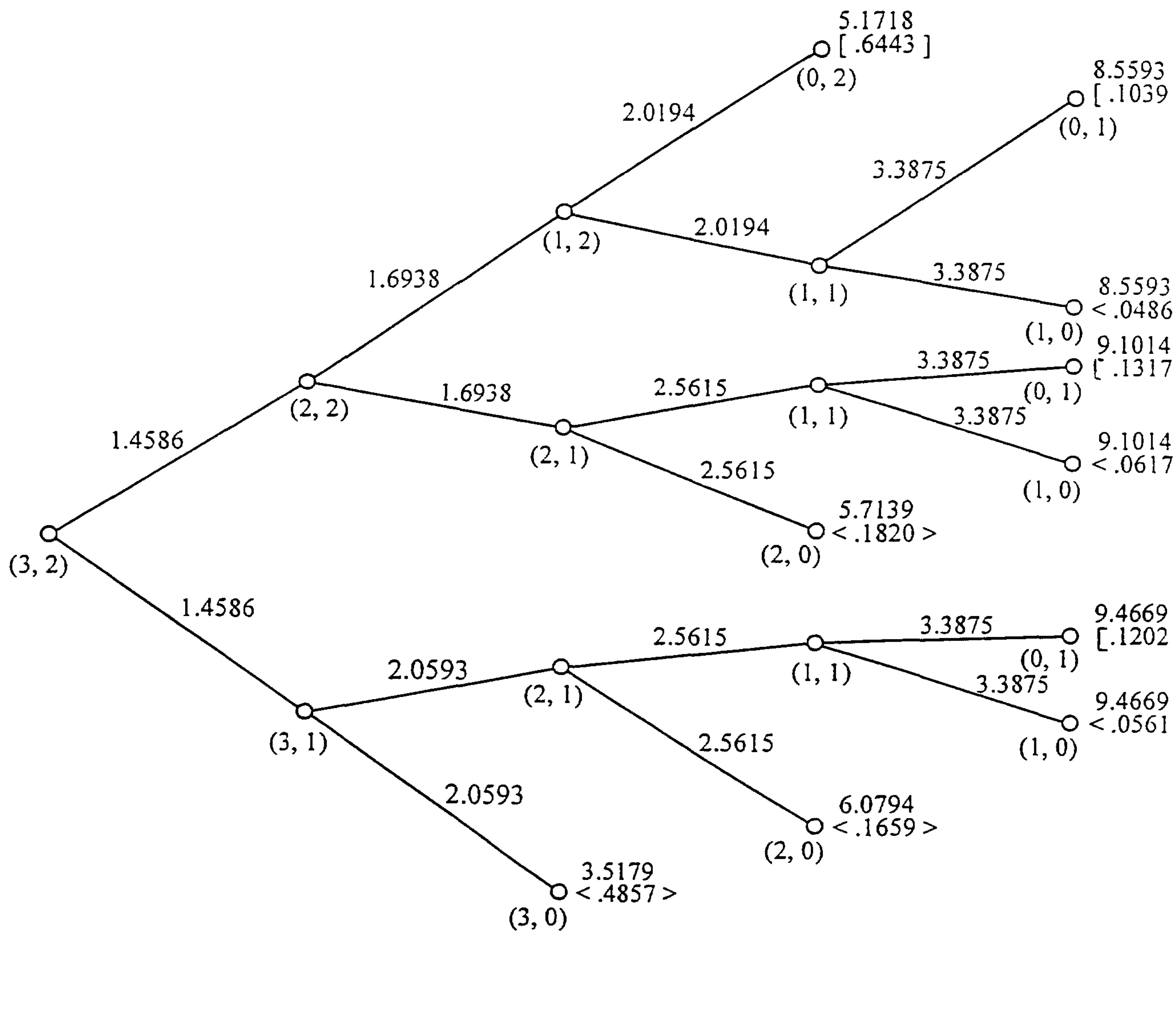

Figure III

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OPERATIONAL ANALYSIS

EXERCISE ORTHODOX APPROACH

PROBLEM 5

1. If the battle ends with Red having defeated Blue, and Red having all three tanks surviving, use the time dependent stochastic equations to determine the probability of this outcome.
2. Use the time dependent stochastic equations to determine the expected duration of the battle which ends with the result given above.

SOLUTIONS FOR PROBLEM 5

1. Reference should be made to OA 824.

   If $P(r,b,t)$ is the probability that at time t after the start of battle the battle is in state (r,b) then the following equations and conditions represent the equivalent time dependent stochastic Lanchester direct-fire model.

   General equation

   $$P'(r,b,t) = -P(r,b,t)(\rho r + \beta b) + P(r,b+1,t)\rho r + P(r+1,b,t)\beta b$$

   Boundary equation

   $$P'(r,0,t) = \rho r\ P(r,1,t)$$

   $$P'(0,b,t) = \beta b\, P(1,b,t)$$

   Boundary conditions

   $$P(r,b,0) = 0; \qquad P(R,B,0) = 1$$

   $$P(R+1,B,t) = 0; \quad P(R,B+1,t) = 0$$

2. In general, if R and B are reasonably small, the above equations may be used iteratively to solve the stochastic, time-dependent Lanchester direct-fire attrition process.

3. If, in the battle for analysis, the battle ends with $r = 3, b = 0$ then the states which must be gone through are $(3,2); (3,1)$ and $(3,0)$. By a straightforward application of the stochastic equations it is possible to determine the probability that the battle is in one of these states at time t.

4. Taking each battle state in turn:

   (a) $\boldsymbol{P(3,2,t)}$

   From the general equation and the boundary conditions we have

$$P\prime(3,2,t) = -P(3,2.t).(3\rho + 2\beta).$$

   Directly this gives

$$P(3,2,t) = e^{-0.6856t}(\text{substituting for } \rho \text{ and } \beta)$$

   (b) $\boldsymbol{P(3,1,t)}$

$$\begin{aligned} P\prime(3,1,t) &= -P(3,1,t)(3\rho + \beta) + P(3,2,t).3\rho \\ &= P(3,1,t).(.4856) + 0.2856e^{-.6856t} \end{aligned}$$

   This is an equation of the form:

$$P\prime(3,1,t) = -b.\ P(3,1,t) + c\ e^{at} \text{ where } \begin{array}{r} a = .8856 \\ b = .4856 \\ c = .2856 \end{array}$$

   To integrate multiply both sides by e^{bt} so giving

$$\frac{d}{dt}\left[P(3,1,t).e^{bt}\right] = c\ e^{(b-a)t}$$

   This expression integrates directly to give

$$P(3,1,t).e^{bt} = \frac{c}{b-a}.e^{(b-a)t} + N$$

   where N is a constant.

   When $t = 0, P(3,1,0) = 0$ and so

$$N = -\frac{c}{b-a}$$

Hence

$$P(3,1,t) = \frac{.2856}{.4856 - .6856}\left\{e^{-.6856t} - e^{-.4856t}\right\}$$
$$= 1.4280\left\{e^{-.4856t} - e^{-.6856t}\right\}$$

(c) $\boldsymbol{P(3,0,t)}$

From the boundary equations we have

$$P\prime(3,0,t) = .2856\, P(3.1,t)$$
$$= .407836\left\{e^{-\ 4856t} - e^{-.6856t}\right\}$$

so after integration

$P(3,0,t) = -0.83986e^{-.4856t} + 0.59486e^{-.6856t} + M$

where M is a constant.

When $t = 0, P(3, 0l0) = 0$, hence $M = 0.2450$.

So, $P(3,0,t) = 0.245o - 0.83986e^{-.4856t} + 0.59486e^{-\ 6856t}$

5. To determine the probability that the battle will end in state $(3,0)$ it is necessary to $t \to \infty$ into the expression for $P(3,0,t)$. This gives a probability of 0.2450, a value that checks with the previous numerical result shown in Figure II.

6. Since $P(3,0,t)$ is the probability of having reached battle state $(3,0)$ by time t it follows that $P\prime(3,0,t)$ is the probability that it takes a time of exactly t to reach this state. Hence the mean time at which battle state $(3,0)$ is reached is given by:

$$\int_0^\infty tP\prime(3,0,t)dt$$

Substituting for $P\prime(3,0,t)$ we have

$$\int_0^\infty tP\prime(3,0,t)dt = \int_0^\infty 0.407836t\left(e^{-.4856t} - e^{-.6856t}\right)dt$$

$$= \frac{0.83986}{0.4856} - \frac{0.59486}{0.6856} = 0.86188$$

The probability that the path $(3,2);(3,1);(3,0)$ occurs in the conflict is 0.2450. Hence the expected duration of the path, if it occurs is

$$\frac{0.86188}{0.2450} = 3.5179$$

This is the same value as shown in Figure III for the same path.

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OPERATIONAL ANALYSIS

EXERCISE OREGON TRAIL

SCENARIO

1. A Blue commander is tasked with defending his country from invasion by a neighbour, Red, who can muster a numerically superior force. The Red attacker is assumed to have the initiative. He may attack in different places and in different strengths. The defender's policy is to delay the enemy as long as possible and in particular to delay the capture of any one of a number of vital targets in the hope that a political solution will be found before one of these targets falls. Blue is to the west of Red; the frontier runs north-south.

2. The following simplified scenario will be considered. The territory divides naturally into two sectors, north (Sector I) and south (Sector II). In each sector the battle is expected to divide into three phases.

   (a) Phase 1. An advance by Red from the frontier to a first boundary line.

   (b) Phase 2. An advance from the first boundary line to a second boundary line.

   (c) Phase 3. An advance from the second boundary line to occupy a vital target in that sector.

3. When any vital target is occupied, the battle ends.

THE RED THREAT

4. Red has a total of 310 units available. An intelligence estimate suggests that Red will adopt one of two plans. In Plan I, 30 units will be used in each sector during Phase 1. In Phase 2 these will be replaced by 60 units in the north and 40 in the south, and in Phase 3 by 120 in the north and 30 in the south. In Plan II the corresponding numbers are: Phase 1, 30 north, 30 south; Phase 2, 40 north, 60 south; Phase 3, 80 north, 70 south.

5. Blue has a total of 100 units available which may be deployed as follows:

   (a) Phase 1. Blue forces will be stationed in each sector between the frontier and the first boundary line, tasked with fighting a mobile delaying battle imposing a delay of at least four days on the enemy advance. When the Red forces reach the first boundary line these Blue forces will be deemed to be exhausted and will break contact. They cannot be used during subsequent phases of the battle. (Note that it is not possible to use the reserve to reinforce Blue units during Phase 1 of the battle.)

   (b) Phase 2. Other Blue forces deployed between the first and second boundary lines then take over, tasked in exactly the same way to impose a delay of at least four days on the enemy advance. The Blue forces may be reinforced in each sector from a common reserve, but any units called from the reserve are expected to be only 80% effective. When the Red forces reach the second boundary line these Blue forces (including the committed reserve) will be deemed to be exhausted and will break contact. They cannot be used during the third phase of the battle.

   (c) Phase 3. Last of all, Blue forces deployed behind the second boundary line take over, tasked with imposing the maximum delay possible before the Red forces seize one of the vital targets. The Blue forces may be reinforced from the common reserve and in this final phase of the battle such reserves are expected to be 90% effective.

DELAY OF RED ADVANCE

6. If x is the total effective Blue force and R is the total Red force in any sector during any phase, then the duration of that particular phase of the battle is assumed to be given by an expression of the form

$$C + Kx/R$$

So the duration is composed of an unopposed delay time, C, plus an additional delay which increases linearly with the engaged force ratio, K being the constant of proportionality.

NOTATION

7. Where there are two suffixes the first refers to the sector and the second to the phase of the battle. Where there are three suffixes the third refers to the Red plan adopted.

   x_{ij} Blue force initially stationed in Sector i to fight during Phase j of the battle

   y_{ijk} Number of Blue units called from the Blue reserve to reinforce x_{ij} if Red adopts Plan k.

   R_{ijk} Red force allocated to Sector i during Phase j according to Plan k.

Suffixes are also used with the other parameters defined in Paragraph 6 and Table 1 gives values for these parameters while Figure 1 illustrates the use of the notation in diagrammatic form.

R values	$R_{111} = 30$	$R_{121} = 60$	$R_{131} = 120$
	$R_{211} = 30$	$R_{221} = 40$	$R_{231} = 30$
	$R_{122} = 30$	$R_{122} = 40$	$R_{132} = 80$
	$R_{212} = 30$	$R_{222} = 60$	$R_{232} = 70$
C values	$C_{11} = 0.6$	$C_{12} = 0.8$	$C_{13} = 0.6$
	$C_{21} = 1.2$	$C_{12} = 1.4$	$C_{23} = 1.6$
K values	$K_{11} = 10.5$	$K_{12} = 8.7$	$K_{13} = 12.4$
	$K_{21} = 12.6$	$K_{22} = 13.5$	$K_{23} = 9.1$

Table 1 Parameter Values

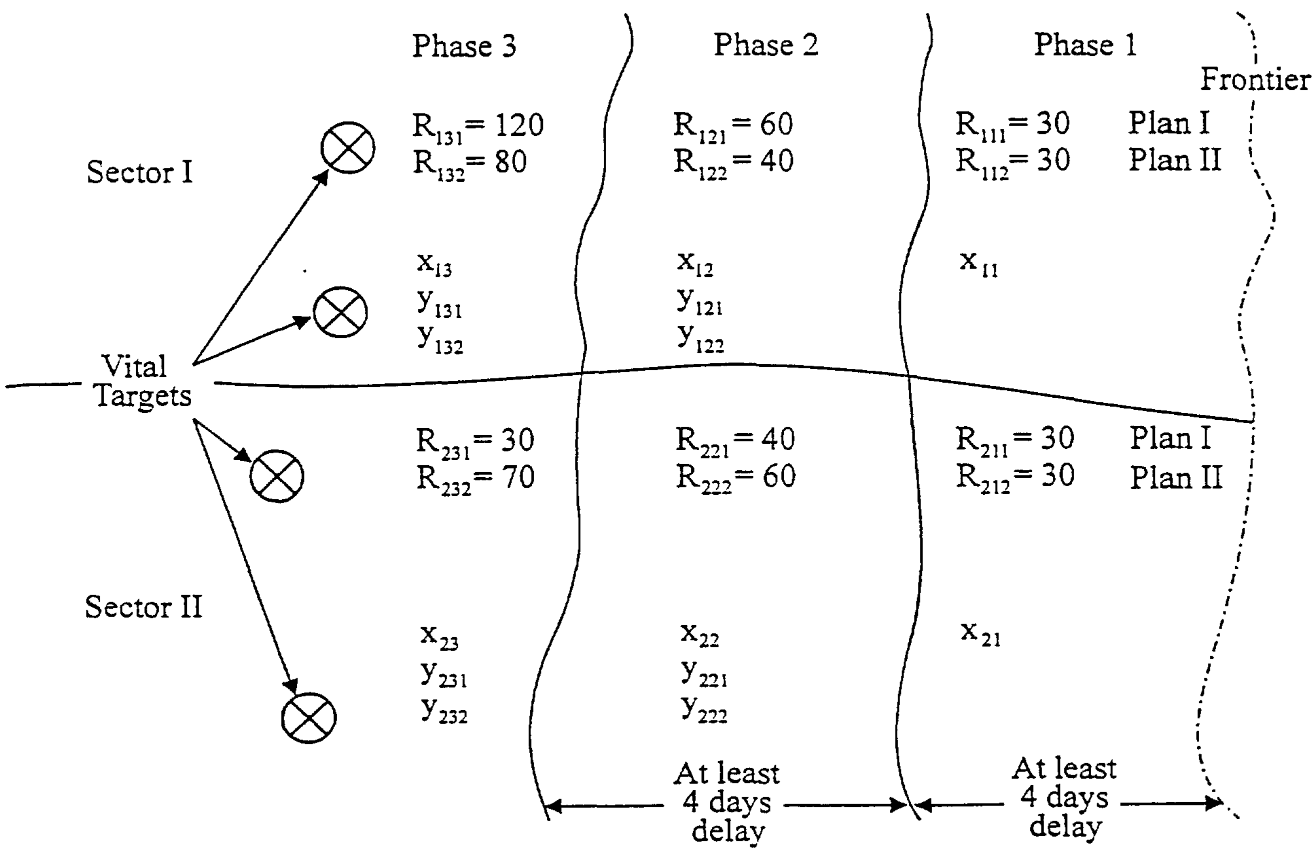

Figure 1 Notation for Problem Formulation

REQUIREMENT

8. A model is required to help address the following questions:

   (a) How should the Blue commander deploy his forces in order to maximise the delay before one of the vital targets falls?

   (b) The number of Units available to the Blue commander is currently 100. How does the solution change if this is increased?

   (c) There is a time delay of at least four days in Phase 1 and 2 in both sectors. How does the solution change if all the time delay restrictions are totally removed?

   (d) How does the deployment of Blue forces change if intelligence sources can say with confidence that Red will operate Plan II?

9. Devise a linear programming model which will meet the requirement.

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OPERATIONAL ANALYSIS

EXERCISE OREGON TRAIL

SUGGESTED SOLUTION

MATHEMATICAL SOLUTION

1. The problem has been deliberately kept simple so as to make its formulation and solution by linear programming techniques possible as an exercise. However it should be noted that more realistic problems can also be tackled in this way.

CONSTRAINTS

2. Phases 1 and 2. A minimum delay of four days is always required for both Phases 1 and 2. For Phase 1 this requires that:

$$C_{i1} + K_{i1}x_{i1}/R_{i1k} \geq 4 \qquad (i, k = 1, 2)$$

Substituting in the parameter values given in Table 1 gives six distinct constraints as follows:

$$\left.\begin{array}{r} 0.6 + 10.5x_{11}/30 \geq 4 \\ 1.2 + 12.6x_{21}/30 \geq 4 \end{array}\right] \text{Phase 1}$$

$$\left.\begin{array}{r} 0.8 + \ 8.7(x_{12} + 0.8y_{121}/60 \geq 4 \\ 0.8 + \ 8.7(x_{12} + 0.8y_{122}/40 \geq 4 \\ 1.4 + 13.5(x_{22} + 0.8y_{221}/40 \geq 4 \\ 1.4 + 13.5(x_{22} + 0.8y_{222}/60 \geq 4 \end{array}\right] \text{Phase 2}$$

3. The Blue force has only 100 units in total to deploy, which gives two further constraints (one for each possible Red plan):

$$\begin{aligned}
x_{11} + x_{12} + x_{13} + x_{21} + x_{22} + x_{23} + y_{121} + y_{131} + y_{221} + y_{231} &= 100 \\
x_{11} + x_{12} + x_{13} + x_{21} + x_{22} + x_{23} + y_{122} + y_{132} + y_{222} + y_{232} &= 100
\end{aligned}$$

4. Total time of battle for each sector and plan. Let t_{ik} be the total time taken before a vital target in Sector i falls when Red adopts Plan k. Then:

$$t_{ik} = \sum_{j=1}^{3} \left[C_{ij} + K_{ij}\left(x_{ij} + a_j y_{ijk}\right)/R_{ijk}\right]$$

where a_j is the effectiveness factor for reserve forces, so that $a_1 = 0$ (no reserve deployment allowed to Phase 1), $a_2 = 0.8$, and $a_3 = 0.9$. This leads to the following four constraints:

$$\begin{aligned}
\left[0.6 + \frac{10.5x_{11}}{30}\right] + \left[0.8 + \frac{8.7\left(x_{12} + 0.8y_{121}\right)}{60}\right] + \left[0.6 + \frac{12.4\left(x_{13} + 0.9y_{131}\right)}{120}\right] &= t_{11} \\
\left[0.6 + \frac{10.5x_{11}}{30}\right] + \left[0.8 + \frac{8.7\left(x_{12} + 0.8y_{122}\right)}{40}\right] + \left[0.6 + \frac{12.4\left(x_{13} + 0.9y_{132}\right)}{80}\right] &= t_{12} \\
\left[1.2 + \frac{12.6x_{21}}{30}\right] + \left[1.4 + \frac{13.5\left(x_{22} + 0.8y_{221}\right)}{40}\right] + \left[1.6 + \frac{9.1\left(x_{23} + 0.9y_{231}\right)}{30}\right] &= t_{21} \\
\left[1.2 + \frac{12.6x_{21}}{30}\right] + \left[1.4 + \frac{13.5\left(x_{22} + 0.8y_{222}\right)}{60}\right] + \left[1.6 + \frac{9.1\left(x_{23} + 0.9y_{232}\right)}{70}\right] &= t_{22}
\end{aligned}$$

OBJECTIVE FUNCTION

5. Blue must adopt a strategy to maximise the delay before the first of the vital targets fall, taking into account the fact that Red has two options: Plan I or Plan II. Hence Blue must look to the worst case (from Blue's perspective) and make that delay as long as possible, on the assumption that Red will be able to exploit that worst case, ie Blue must adopt a max-min strategy. Thus the objective function for Blue is to maximise the minimum of the four possible durations t_{11}, t_{12}, t_{21} and t_{22}. Mathematically:

$$\text{Maximise } Z = \min(t_{11}, t_{12}, t_{21} \text{ and } t_{22})$$

6. This may be converted to standard linear programming format by adding four new constraints and a new variable t, such that:

$$t \leq t_{11},\ t \leq t_{12},\ t \leq t_{21} \text{ and } t \leq t_{22}$$

with the objective function:

$$\text{Maximise } Z = t$$

LINEAR PROGRAMMING FORMULATION

7. Putting all the constraints together and simplifying as far as possible leads to the following linear programming formulation of the problem:

Maximise $Z = t$

subject to:

$x_{11} \geq 9.71$

$x_{21} \geq 6.67$

$x_{12} + 0.8y_{121} \geq 22.07$

$x_{12} + 0.8y_{122} \geq 14.71$

$x_{22} + 0.8y_{221} \geq 7.7$

$x_{22} + 0.8y_{222} \geq 11.56$

$-0.35x_{11} - 0.145x_{12} - 0.103x_{13} - 0.116y_{121} - 0.093y_{131} + t \leq 2$

$-0.34x_{11} - 0.218x_{12} - 0.155x_{13} - 0.174y_{122} - 0.140y_{132} + t \leq 2$

$-0.42x_{21} - 0.337x_{22} - 0.303x_{23} - 0.27y_{221} - 0.273y_{231} + t \leq 4.2$

$-0.42x_{21} - 0.225x_{22} - 0.13x_{23} - 0.18y_{222} - 0.117y_{232} + t \leq 4.2$

$x_{11} + x_{12} + x_{13} + x_{21} + x_{22} + x_{23} + y_{121} + y_{131} + y_{221} + y_{231} = 100$

$x_{11} + x_{12} + x_{13} + x_{21} + x_{22} + x_{23} + y_{122} + y_{132} + y_{222} + y_{232} = 100$

All variables ≥ 0.

OPTIMAL SOLUTION

8. Treating the x_{ij} as real variables gives the following solution:

$$\begin{array}{lll} x_{11} = 39.85, & x_{12} = 18.22, & x_{13} = 0 \\ x_{21} = 29.41, & x_{22} = 7.70, & x_{23} = 0 \\ y_{121} = 4.81 & y_{222} = 4.81 & \end{array}$$

$$y_{122} = y_{131} = y_{132} = y_{221} = y_{231} = y_{232} = 0$$

$$t = 19.15 \text{ days}$$

9. In practice it is probable that Blue will have to deploy complete units, so that the problem must be treated as an integer programming problem. The integer solution which is illustrated diagrammatically in Figure 2 is:

$$x_{11} = 39, \quad x_{12} = 19, \quad x_{13} = 0 \text{ units}$$
$$x_{21} = 29, \quad x_{22} = 8, \quad x_{23} = 0 \text{ units}$$

Reserve = 5 units (to be deployed in Phase 2 to Sector I or II as appropriate). Delay until a vital target falls = 19 days.

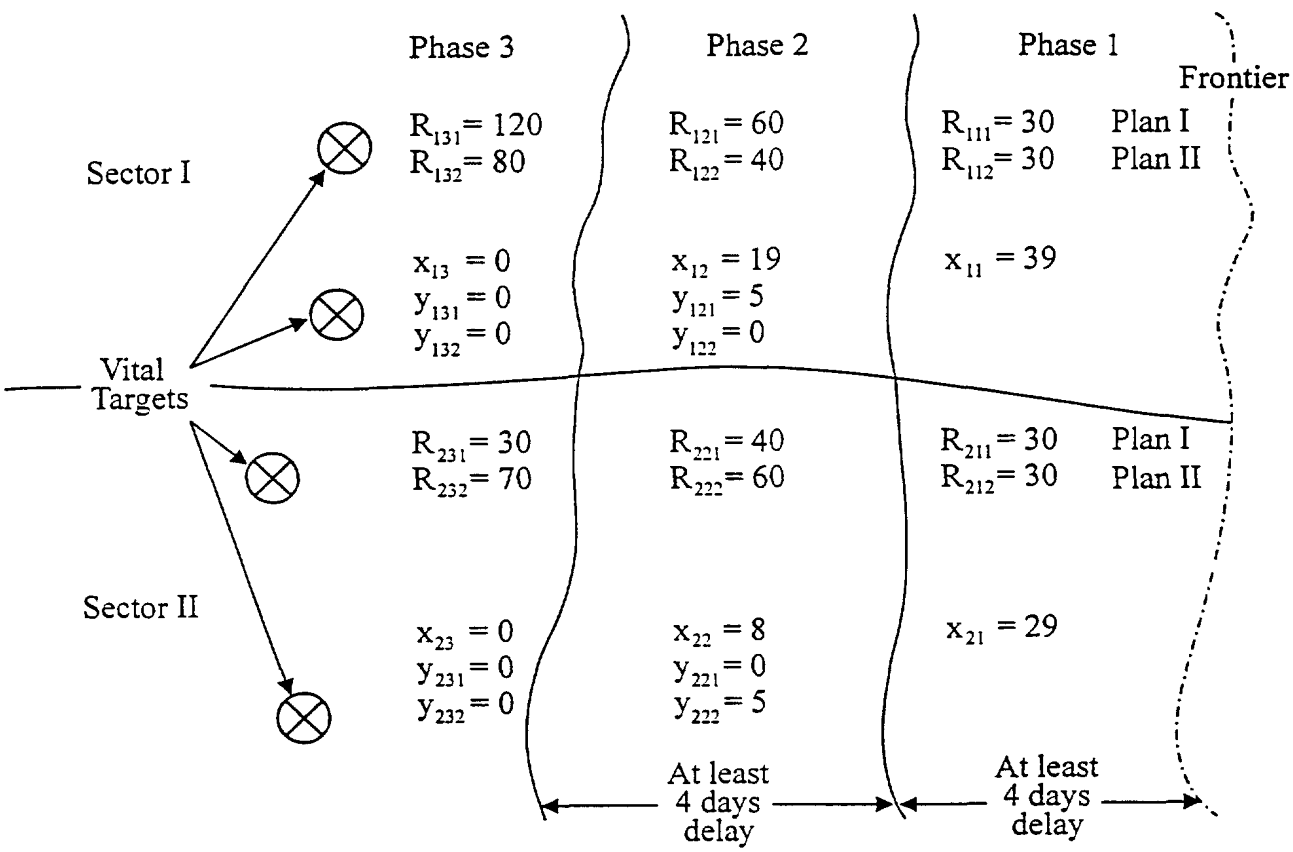

Figure 2 Optimal Solution (Total Delay = 19 Days)

SENSITIVITY ANALYSIS

10. The solution to the problem as originally stated is given again in Table 2 and compared with results from various sensitivity runs of the model. The analysis examines the effect of:

    (a) Increasing the total Blue force.

    (b) Removing the restriction of four days minimum delay time in Phases 1 and 2.

    (c) Making Plan I the same as Plan II (equivalent to Blue having intelligence information that Red will adopt Plan II).

Formulation	Total Delay (Days)	Blue Force Deployment (units)					
		x_{11}	x_{12}	x_{21}	x_{22}	y_{121}	y_{222}
Original	19.00	39	19	29	8	5	5
a. Increasing Blue force							
150 units	28.44	66	19	52	8	5	5
200 units	38.03	94	19	74	9	4	4
250 units	47.64	121	19	97	8	5	5
300 units	57.14	148	19	120	8	5	5
b. Removing 4-day delay							
100 units	21.95	57	0	43	0	0	0
150 units	31.50	85	0	65	0	0	0
200 units	41.16	112	0	88	0	0	0
c. Plan I = II	19.92	42	15	31	12	0	0

Table 2 Sensitivity Analysis Results

11. It is evident that the solution to the original problem is to allocate just enough Blue forces to Phase 2 and to the reserve to satisfy the requirement for a minimum delay of four days in Phase 2 of the battle. The remainder of the Blue force is then allocated to Phase 1 of the battle. If the four-day requirement is dropped then the whole of the Blue force is deployed forward for Phase 1 of the battle and no reserve is maintained.

12. It should be emphasised that the solution is crucially dependent on the assumptions and on the parameter values specified in Table 1 and no sensitivity results for variations in those values have been given. The R values in Table 1 reflect how accurately the Red plans are known, while the C and K values include the effects of terrain on the battle through the delays imposed on the enemy advance. It goes without saying that no general conclusions should be drawn from the purely fictitious data and results used in this exercise

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This author attended the War Gaming and Combat Modelling Course from 15 to 19 May 2000. The course is offered by the Applied Math and Operational Research Group at the Royal Military College of Science in Shrivenham, UK. The course syllabus included the deterministic Lanchester Square and Linear Laws, a stochastic Lanchester approach to simulation, measures of effectiveness, exercises for the students, a presentation of several combat models in use or under development today in the UK. This course was an excellent introduction to the fields of war gaming and combat modelling. It is recommended that all junior defence scientists be encouraged to attend this course.

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